Photorefractive Spatial Solitons

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1 Introduction

Photorefractive crystals are electro-optic dielectrics that host a small amount of photosensitive impurities. Light propagation leads to the generation of out-of-equilibrium mobile charge, that, in order to reach a stable electro-static configuration, redistributes throughout the crystal. The ensuing space-charge field, modifying electro-optically the crystal index of refraction, changes the trajectory of the ionizing light distribution, altering, in turn, the original charge-equilibrium conditions [1]. This feedback mechanism gives rise to a variety of nonlinear effects that go under the generic term of photorefractive nonlinear optics [2] [3] [4] [5]. For confined optical beams, nonlinear beam dynamics leads to two basic qualitatively different phenomena: beam fanning and self-lensing, connected, respectively, to the two basic charge transport mechanisms, diffusion and drift. Photorefractive spatial solitons emerge when beam self-focusing exactly balances diffraction, and are thus generally connected to regimes in which charge drift plays a fundamental role [6] [7] [8][9]. In a nonlinear beam perspective, spatial beams self-trap when the light-space-charge feedback mechanism finds its dynamic equilibrium point in a nondiffracting slab or needle of light, corresponding to an appropriate waveguide-like refractive index distribution. In what follows, we discuss such nonlinear phenomena, concentrating in particular on the basic theory and phenomenology.

2 Photorefractive Beam Nonlinearity

Photorefractive beam dynamics are fruit of the combined effect of optical wave-propagation and electro-optic response to the electric field generated by dislocation of photo-excited charge. A description starts from two basic issues: 1) Given an optical distribution, what is the induced equilibrium charge dislocation and hence the photoinduced electric field? 2) How does this electric field modify, through the electro-optic response, optical propagation?

Space-charge Field

The basic model that describes charge separation in a photorefractive was formulated by Kukhtarev and Odoulov [10]. Although it makes use of a number of approximations, it incorporates all the basic ingredients that allow for the prediction and description of photorefractive self-trapping, at least in the case of one-dimensional beams, that is, beams that are confined in only one transverse dimension. Light-matter interaction reflects the typical band-structure of a lightly doped dielectric. In particular, the structure can normally be approximated by considering two intraband levels: one donor and one acceptor. For n-type photorefractives, donor sites can be optically ionized by light of an appropriate wavelength, generally visible, depending on the given impurity. Furthermore, the concentration of donor sites N_d ($N_d \approx 10^{18}$-$10^{19}\mathrm{cm}^{-3}$) is much greater than that of acceptor impurities N_a (i.e. $\alpha \equiv N_d/N_a \gg 1$). In absence of ionizing illumination and thermal excitation, equilibrium is reached when all the acceptor sites are ionized by donors, that is, the concentration of ionized donors N_d^+=N_a. For a given optical intensity distribution $I(x, y, z)$, charge equilibrium is reached when mobile charge generation and recombination exactly balance, that is $(\beta$+$sI)(N_d$-$N_d^+)$=$\gamma N N_d^+$, where β is the thermal excitation rate, s is the photoexcitation cross-section, γ is electron recombination rate, and N is the local concentration of conduction electrons. In general, the non-equilibrium condition is described by the rate equation

$$\frac{\partial}{\partial t} N_d^+ = (\beta + sI)(N_d - N_d^+) - \gamma N N_d^+. \tag{1}$$

Electrons move in the crystal under the influence of drift, diffusion and, in some noncentrosymmetric samples, the photovoltaic effect [11], giving rise to a current density $\boldsymbol{J}$ described by

$$\boldsymbol{J} = q\mu N \boldsymbol{E} + k_b T \mu \boldsymbol{\nabla} N + \beta_{ph}(N_d - N_d^+) I \boldsymbol{c} \tag{2}$$

where -q is the electron charge, μ is the charge mobility, k_b is the Boltzmann constant, T is the crystal temperature, and β_{ph} is the component of the photovoltaic tensor along the optical axis $\boldsymbol{c}$. The model is completed by imposing charge continuity and the Poisson equation, along with appropriate boundary conditions,

$$\frac{\partial}{\partial t}\rho + \nabla \cdot \boldsymbol{J} = 0 \tag{3}$$

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon} \tag{4}$$

where ρ=$q(N_d^+$-N_a-$N)$, and ϵ is the crystal dielectric constant. For conditions in which the electro-optic sample is biased by a constant applied voltage V, the condition

$$V = -\int_a^b \boldsymbol{E} \cdot d\ell \tag{5}$$

holds, where the line integral goes from one electrode (a) to the other (b). Finally,

$$\nabla \times \boldsymbol{E} = 0. \tag{6}$$

This system of equations can be cast into a single nonlinear differential equation relating $\boldsymbol{E}$ to I [12].

Two fundamental photorefractive time scales emerge: the charge recombination time (or charge lifetime) τ_r=1/($N\gamma$) (see eq.(1)), and the dielectric relaxation time τ_d=ϵ/($\mu N q$) (see eq.(3)). For most configurations of interest, $\tau_r \ll \tau_d$, and the study of space-charge evolution starts by considering adiabatically $\partial N_d^+/\partial t$=0 in eq.(1), along with the generally valid assumptions that $N \ll N_a$, and the mentioned $\alpha \gg 1$. The resulting nonlinear equation relating $\boldsymbol{E}$=$\boldsymbol{E}(I)$ is

$$\nabla \cdot [\frac{\gamma\epsilon}{q\mu s\alpha}\frac{\partial \boldsymbol{E}}{\partial t} + \boldsymbol{E}(\beta/s+\ I\ )\frac{1-\frac{\epsilon\nabla\cdot\boldsymbol{E}}{\alpha N_a q}}{1+\frac{\epsilon\nabla\cdot\boldsymbol{E}}{N_a q}} + \\ + \frac{k_b T}{q}\ \nabla \cdot \left((\beta/s+I)\frac{1-\frac{\epsilon\nabla\cdot\boldsymbol{E}}{\alpha N_a q}}{1+\frac{\epsilon\nabla\cdot\boldsymbol{E}}{N_a q}}\right)] = 0.$$

Under appropriate approximations, this equation allows one to calculate $\boldsymbol{E}$=$\boldsymbol{E}(I)$.

Beam Propagation

The space-charge field $\boldsymbol{E}$ influences beam propagation through the electro-optic modulation of the index of refraction. As in standard electro-optic samples, the index modulation can be phenomenologically described by the relation

$$\Delta n_{ij} = -\frac{1}{2}n^3 r_{ijk}E_k - \frac{1}{2}n^3\epsilon^2 g_{ijkl}E_k E_l \tag{7}$$

where n is the unperturbed crystal index of refraction, r_{ijk} and g_{ijkl} are respectively, the linear and quadratic electro-optic tensors, and $\boldsymbol{E}$=(E_x, E_y, E_z). For a noncentrosymmetric sample, the quadratic term is generally irrelevant,whereas for centrosymmetric ones, the linear response is absent.

For a monochromatic paraxial beam, propagation is described by the parabolic equation

$$\left[\frac{\partial}{\partial z} - \frac{i}{2k}\nabla_\perp^2\right] A_i(x,y,z) = -\frac{ik}{n}\Delta n_{ij}A_j(x,y,z) \tag{8}$$

where k=2πn/λ is the wave-vector, A_x and A_y are the transverse components of the slowly varying optical field, i.e. $\boldsymbol{E}_{op}(\boldsymbol{r}$,t)=$\boldsymbol{A}(\boldsymbol{r})$exp($ikz$-$i\omega t$)

$\omega = 2\pi c/n\lambda$, and I=$|\boldsymbol{A}(\boldsymbol{r})|^2$. Through Eq.(7), and the relationship $\boldsymbol{E}$=$\boldsymbol{E}(I)$, Eq.(8) becomes the general nonlinear equation that describes almost all photorefractive nonlinear beam dynamics, and, in particular, solitons.

Photorefraction is thus a consequence of interplay bewteen the small absorbed (i.e. non-propagating) part of an optical beam and the remnant propagating light, that, in fact, propagates *linearly* in the medium (like in thermo-optic effects). This fact, that distinguishes it from the fundamental nonlinear optical phenomena connected to direct optical self-action, is useful in understanding both the strengths and the limits of photorefraction. The strengths lie in the fact that the response is due to a temporally extended buildup process, allowing for the observation of intense beam self-action even with low optical intensities. This temporally "nonlocal" response, however, limits attainable dynamics. Thus, photorefractive phenomena are sometimes referred to as *beam* nonlinearities instead of *optical* nonlinearities, this characteristic being also at the basis of phenomena connected to self-trapping of time-varying light beams.

3 Self-trapping Mechanisms

The highly nonlinear system described above gives rise to a host of different phenomena associated with beam self-trapping, which have been the object of intense investigation during the past decade. The starting point of the associated scientific effort is the idea, formulated in 1992 [6] [13] and later confirmed in pioneering experiments [14], that, in biased drift-dominated photorefractives, self-lensing could lead to trapping of confined optical beams. In what follows we shall describe the various different types of photorefractive self-trapping mechanisms, i.e. quasi-steady-state solitons, steady-state screening solitons, photovoltaic solitons, semiconductor screening solitons, and centrosymmetric self-trapping. Attention is concentrated on scalar configurations, although effects connected to the tensorial nature of eq.(7) have been investigated [15] [16].

3.1 Quasi-Steady-State Solitons

Quasi-steady-state solitons are beams that self-trap in a biased photorefractive crystal during a finite time window, and subsequently undergo beam diffraction, break-up and fanning [14]. They contain, in an embryonic form, most of the ingredients that lead to the formation of stable self-trapping. They were the first type of photorefractive spatial soliton to be predicted [6] [13] and observed, both as bright and dark self-trapped beams in one and two-transverse dimensions [14] [17] [18], and are still today subject of investigation [19] [20] . Given their transient nature, they have been mostly associated to studies on the time evolution of photorefractive beam dynamics [19][21][22] [23], taking into account the full time-dependent model of Eqs.(1-6). Their potential for passive optical guiding in bulk media was recognized early on [24], in an experiment that represents the first reported application of photorefractive solitons, in which a dark photorefractive quasi-steady-state soliton was made to guide a non-photoactive read beam, as shown in Fig.(1).

Fig. 1. Top view photographs of (a) a dark 1+1D quasi-steady-state 12μm soliton at λ=457nm (b) a linearly diffracting dark notch with drift nonlinearity disactivated (c) a guided λ=632nm He-Ne beam and (d) the same undergoing diffraction when no soliton is present (from ref.([24]))

Although the theoretical description of these self-trapped beams is rather intricate, the physical mechanism in the slab soliton case is intuitive. As schematically illustrated in Fig.(2), bright slab-quasi-steady-state solitons are generated when a continuos-wave visible photoactive laser beam (i.e., capable of ionizing photorefractive impurities) is focused onto the input facet of a zero-cut photorefractive sample (for example, a doped SBN crystal).

Fig. 2. Schematic set-up allowing the generation and observation of quasi-steady-state photorefractive bright slab solitons. Dark solitons are generated with an appropriate phase mask at the input, and 2+1D needle solitons are generated substituting the cylindrical lens with a spherical one.

The beam is made to propagate along an ordinary principal axis (i.e., the z axis), being itself extraordinarily polarized (i.e., along the x axis). Electrodes on the crystal x facets allow the application of the external bias field. The beam is confined in the x dimension, whereas it is extended (nondiffracting) in the second transverse direction y. During the initial stages of beam evolution, for times shorter than τ_d , mobile charges are photoexcited and drift in the external field. For n-type samples, such as SBN, mobile electrons statistically drift towards the positive electrode and the ensuing charge separation with the locked ionized impurities gives rise to a double layer that screens the external field. The electro-optic response of Eq.(7) is reduced to the scalar equivalent

$$\Delta n = -\frac{1}{2} n^3 r_{33} E \tag{9}$$

where $i = j = k = 3$ (x axis) and r_{33} is the contracted form of r_{333}. For an appropriate arrangement, $r_{33}E > 0$, and the unscreened region of the crystal suffers a global decrease of index of refraction. In the screened region, this effect is weaker, and the net result is a higher index of refraction in the illuminated region, giving rise to self-lensing. During the screening stages, if the applied field is sufficiently high, the charge pattern passes through a self-lensing regime such as to trap the diffracting beam into a slab-soliton. However, the presence of propagating light continues generating mobile charges, and these keep drifting towards the positive pole until the actual charge separation reaches saturation when screening is total. This leads to a highly insensitive saturated response in *all* illuminated regions, giving rise to a generally widened index modulation (a wide step-index "waveguide") that does not correspond to a self-trapping index structure. This simple picture holds for beam intensities much higher than typical dark equivalent illuminations, i.e., $I \gg I_d \equiv \beta/s$ (of the order of 10^{-3}-10^{-6}W/cm^2), and for applied external fields much higher than typical diffusion fields ($\sim 10^2$ V/cm for a 10 μm beam). The quasi-steady-state self-trapping regime has been analytically described in refs.([6] [13] [25]) by means of a phenomenological model, whereas numerical investigation of the entire transient regime in both 1+1D and 2+1D can be found in refs.([19][21] [22] [23]). An analytical expression of the time dependent space-charge field can furthermore be found in refs.([12] [26]). Both theory and experiment agree on two main features: (1) quasi-steady-state soliton formation is not dependent on the intensity of the propagating beam (as long as $I \gg I_d$) and (2) quasi-steady-state solitons are characterized by a whole *region* of existence, forming, for a given input beam size, in a range of external bias fields (always being $V/L \gg E_d$)[27][28].

Fig. 3. Numerical simulation results for the normalized space-charge field E evolution for t/T$_e$=0, 0.1, 0.2, 0.5, 1, 2, 5, 10, ∞, with T$_e$=$\gamma\epsilon$/qμsα (left) and a typical half-width-half-maximum (HWHM in normalized units) evolution for a 1+1D soliton configuration (from ref.([21]))

As can be seen in Fig.(3) (on the right), the quasi-steady-state regime occurs for a relatively extended plateau (note the logarithmic time scale in Fig.(3)). The plateau arises when the intermediate light trapping space-charge field E forms, in a process that can be qualitatively explained as follows: Given that time-dynamics are locally proportional to light intensity, once trapping occurs, space-charge broadening is temporarily slowed down by the absence of light in the immediate vicinity of the screening region. As trapping is weakened, time dynamics speed up and reach a time-behaviour similar to the build-up process.

3.2 Screening Solitons

The most widely studied self-trapping mechanism in photorefractives is without doubt the so-called screening nonlinearity. The reason for this lies in the fact that the resulting self-trapped beams are steady-state, that is, they do not form and break-up in a transient, but rather persist in time as long as the laboratory parameters maintain their values. Furthermore, in their one-dimensional manifestation as 1+1D slab solitons, the model described leads, under appropriate approximations, to an explicit saturable Kerr-like nonlinear equation (in a merely formal analogy), that allows direct qualitative and quantitative prediction and interpretation, as opposed to either merely phenomenological models or purely numerical strategies. Screening solitons are supported by a slightly modified quasi-steady-state soliton-supporting configuration, and, like these, exist also in the higher dimensional needle case. The higher dimensionality, of central import for beam steering applications, fundamentally complicates the theoretical description, and raises issues connected to the anisotropy of the underlying physical mechanism.

The discovery of steady-state photorefractive screening solitons began with the preliminary observation that response saturation bringing to soliton annihilation in quasi-steady-state configurations can be inhibited making use of an artificial background illumination [29]. It was soon shown that this could indeed lead to stable self-trapped photorefractive beams [30] [31] [32]. The basic idea is the following: Since the runaway charge separation leading to quasi-steady-state soliton decay is due to the inevitable charge accumulation at the edge of the propagating beam, this being a direct consequence of charge recombination and low dark conductivity, by artificially increasing the global crystal conductivity with a constant optical illumination of the whole sample, charge can move through the equivalent circuit formed by the crystal and the voltage supply, thereby avoiding unrestrained buildup. Final charge separation depends on the ratio of the intensity of the propagating beam to the background illumination, and the resulting electric field in the crystal follows qualitatively that of a series of resistors to which a constant voltage V is applied, resulting lower in the beam region, and thus, like in the previous quasi-steady-state case, giving rise to self-lensing [33]. Since the mechanism is based on a dynamic equilibrium, stable steady-state spatial self-trapping occurs when this mechanism generates the self-consistent charge separation that exactly balances optical diffraction. This occurs for a precise set of physical parameters, and gives rise to what is generally referred to as the "soliton" existence curve [34], making screening steady-state solitons very different from their transient counterparts.

Screening solitons have been observed in a number of different configurations, attesting to their relatively general nature. They have been observed as bright 1+1D slab solitons [29], dark 1+1D slabs [30] [35], as bright 2+1D needle solitons [36] [37], and even as dark needles [39]. They have been detected in different crystals, such as SBN [33] [36] [37] , in BSO and BGO [29] [38], in semiconductor InP [40] [41], in $BaTiO_3$ [42], and have been predicted and observed in ferroelectrics in the high symmetry paraelectric phase [43] [44] [45] .

The experimental apparatus that allows screening soliton formation is in all similar to that needed to observe quasi-steady-state solitons illustrated in Fig.(2), with the addition of a background illumination, as indicated in Fig.(4). In the most appropriate configuration, the background illumination is obtained by illuminating the entire crystal with a plane wave of the same wavelength λ as that of the soliton beam, polarized orthogonal to the soliton beam polarization, i.e., orthogonal to the crystal c axis (ordinary beam), and made to copropagate along the z axis, with the soliton beam itself. Use of the same wavelength allows the implementation of a single CW laser for the entire setup, and makes the theoretical interpretation independent of the sample specific optical photorefractive cross-section s=s(λ). Ordinary polarization avoids the possible coupling of background light into the soliton supporting index pattern, given that the off-axis tensorial terms of the electro-optic response described in Eq.(7) are generally weaker than diagonal ones (i.e. $r_{13} < r_{33}$). Finally, copropagation makes the ratio of the peak soliton beam intensity I_0 to the background illumination I_b constant for each value of z along the propagation direction, given that the absorption is the same for the two beams. This last condition is particularly important because soliton formation strongly depends on the value of this ratio $u_0^2 = I_0/I_b$, known as the intensity ratio, that must be independent of z.

Fig. 4. Schematic set-up allowing the generation and observation of steady-state photorefractive bright screening needle solitons. Note the introduction of a background illumination and a selector before the z-axis CCD camera.

Fig. 5. Dark slab screening solitons. Beam profiles and photographs of the input, normally diffracting output, and soliton output beams after propagation in a 5-mm SBN crystal. Taken from [35]

Fig. 6. Needle Soliton: Photographs and beam profiles (cross sections) of (a) a c-polarized 488-nm needle soliton beam and a (b) c-polarized 632.8-nm 2D non-photoactive probe beam. Top photographs and sections show input beams, the middle show the diffracted beams at zero voltage, and the bottom show the slightly distorted needle soliton output and the corresponding passively guided probe beam. Taken from [51]

Screening solitons have been used to observe a number of startling nonlinear effects, such as soliton spiralling [46], fusion [47], and soliton annihiltaion [48], to realize new optical devices, such a reconfigurable direction coupler [49] and an enhanced second-harmonic generation [50] [51], and to investigate hereto unexplored phenomena, such as self-trapping of incoherent light beams, discussed in Chapter... .

The theoretical description of slab screening solitons [30] shown in Fig.(5) is to be considered one of the most important successes of the Kukhtarev model described above, whereas the description of needle solitons shown in Fig.(6) is still subject of investigation, and the strikingly symmetric self-trapping that emerges from a highly asymmetric physical process baffles more than a researcher [52] [53] [54] [55].

Screening Slab-Solitons Screening slab-soliton description was formulated on the basis of an approximate reduction of the full Kukhtarev model [30] [31] [32] [33] [34]. Consider the reduced 1+1D system associated with steady-state slab-solitons. This reduces the relationship giving the electric field E to a partial differential equation in $|\boldsymbol{A}(x,z)|^2$ and $I(x,z) = \boldsymbol{A}(x,z)$. Furthermore, the actual typical values of the physical quantities involved are such that $\frac{\varepsilon \nabla \cdot E}{\alpha N_a q} \ll 1$, whereas in most cases $\gamma \equiv \frac{\varepsilon \nabla \cdot E}{N_a q} \ll 1$ is a much stronger, but still relatively plausible, assumption. For example, in SBN, a 10μm screening soliton is formed with an electric field scale $|\boldsymbol{E}| \sim 10^2$ kV/m, $\mathrm{N}_a \approx 10^{22}$ m^{-3}, $\gamma \sim 0.1$. Neglecting thus γ with respect to one (and even more so γ/α), the nonlinear differential equation relating the optical field to the space-charge field reads

$$\frac{\partial}{\partial x}\left(E\left(I_b+I\right)+\frac{k_b T}{q}\frac{\partial}{\partial x}\left(I_b+I\right)\right)=0, \tag{10}$$

where the z derivatives have been neglected with respect to x ones, a valid assumption for paraxial cases, and, in particular, if we are looking for soliton solutions. The second term on the LHS comes directly from the diffusion term in eq.(2), assuming $\beta_{ph} \equiv 0$. We can neglect it in a drift dominated regime, since in general, $k_b T/(q\ell) \ll |E|$, where ℓ is the transverse spatial soliton scale (10 μm in the above mentioned case). The space-charge field thus approximately obeys the relationship

$$E=\frac{\delta}{(I_b+I)}\simeq -\frac{V}{L}\frac{1}{\left(1+\frac{I}{I_b}\right)} \tag{11}$$

where δ is a constant that is fixed by the boundary condition of eq.(5). Given the fact that $\ell \ll L$, where L is the distance between the crystal electrodes (sample size in the transverse x direction), $\delta \simeq -I_b V/L$. Given the scalar electro-optic response of eq.(9) in a ferroelectric (neglecting the quadratic term), the nonlinearity is of the type $\Delta n \sim 1/\left(1+\frac{I}{I_b}\right)$, i.e., a saturated Kerr-like nonlinearity. The nonlinear propagation equation is obtained by substituting eq.(11) into eq.(9), and then substituting this expression for Δn directly into the parabolic equation eq.(8). The final nonlinear scalar propagation equation reads

$$\frac{d^2u(\xi)}{d\xi^2}=\pm\left(\frac{\Gamma}{b}-\frac{1}{1+u(\xi)^2}\right)u(\xi) \tag{12}$$

where we have imposed self-consistently the scalar solitary-wave solution form $A(x,z) = u(x)e^{i\Gamma z}\sqrt{I_b}$, and have normalized the transverse spatial scale to the so-called nonlinear length scale $d = (\pm 2kb)^{-1/2}$, i.e., $\xi = x/d$, with $b = (1/2)kn^2 r_{33}(V/L)$. The plus sign corresponds to $b > 0$, and leads to self-focusing and bright spatial solitons, whereas the minus sign corresponds to $b < 0$, and describes self-defocusing and dark spatial solitons. Equation (12) lacks a first derivative, and can be integrated once, giving the relationship $\Gamma/b = \log\left(1+u_0^2\right)/u_0^2$ for bright beams, and $\Gamma/b = 1/\left(1+u_\infty^2\right)$ for dark, where $u_\infty = u(\infty) = -u(-\infty)$. It is not solvable analytically, unless $u_0 \ll 1$, where it reduces to a Kerr nonlinearity, but refers to a situation in which solitons are not observable . In general, it can be solved numerically and examples of self-trapped beam profiles are reported, for example, in ref.[12]. Profiles are more similar to a hyberbolic secant function than a Gaussian, this meaning that in experiments, in an initial evolution, the Gaussian laser beam is adiabatically transformed into a stable soliton profile. One important issue is the so-called soliton existence curve [34]. For a given value of u_0, the self-trapped profile has a given width, in terms of the normalized spatial coordinate ξ. This means, experimentally, that for a given input Gaussian FWHM and a given value of u_0, there is a value of V that allows the observation of a stable screening soliton [33].

Whereas the qualitative success of this treatment is evident, quantitative agreement is far from trivial. Experiments aimed at drawing a quantitative comparison between theory and experiment indicate that agreement is not full, and this is generally attributed to a series of factors [34]. First of all, the theory is an analytical approximation of the photorefractive process. Secondly, there is evidence that the small tensorial electro-optic coupling of non-diagonal terms (see eq.(7)) cannot be wholly neglected, and some of the background illumination interacts with the soliton supporting pattern, changing beam trapping conditions. Lastly, as occurs for most experiments involving photorefractive ferroelectrics, the electro-optic parameters vary from sample to sample, depending even on the type of crystal configuration, making comparison ardous [2] [3]. Yet another complication, or rather, area of research still to be explored, is represented by transverse instability [56].

Screening Needle-Solitons One of the major breakthroughs of nonlinear photorefractive beam dynamics is the steady-state self-trapping of two-dimensional beams (2+1D), leading to diffractioless propagation of micron-size light needles [36] [37], or more generically, two-dimensional waves, such as vortexes [39]. These have been widely documented in various crystals [43][51] and configurations, in strict analogy to their one-dimensional counterparts. Most results indicate that self-trapped beams originating from the focused fundamental circular symmetric TEM_{00} mode of a laser are to a good approximation themselves circular symmetric [57], and that a similar symmetry conservation seems to hold also for dark solitons [39]. This experimental fact does, however, represents a theoretical riddle [26] [58]. Even only from an intuitive point of view, there is no apparent reason why a circular-symmetric beam should self-focus maintaining this symmetry, when the very nonlinearity that seems to allow self-trapping is altogether not symmetric. There are two main asymmetries embedded in the physical process. The first is the electro-optic response, that is highly anisotropic. The second is the direction of the bias electric field, that is directed initially along the x direction, but undergoes inevitable distortion during charge separation in the full-2D transverse photorefractive mechanism. This riddling situation is even more startling since some experimental evidence indicates strong asymmetric effects [59]. The difficulty in understanding the intuitive basis of needle self-trapping is not mitigated by any sort of quasi-analytical approach, as in the reduced slab-soliton case. In fact, in the 1+1D case, the propagation equation is *local* (see eq.(12)), the non-local nature being contained only in the imposition of the global voltage drop across the crystal (see eq.(5)). The full 2+1D propagation equation is highly non-local, in the sense that the nonlinear local (symmetric) and non-local (asymmetric) interaction terms are comparable. In the already mentioned Kerr-limit of $u^2 \ll 1$, the local term dominates over the non-local one and it is possible to prove the existence of circularly-symmetric bright and dark solitons and to derive the corresponding existence curve [26]. This regime, however, cannot be experimentally explored, the accessible one being that for which $u^2 > 1$. In this important saturated regime, the analytical approach becomes extremely complicated and the nonlinear wave-equation does not lend itself to any analytical solution or approach. All that can really be said is that, from a purely mathematical point of view, exact circularly-symmetric solutions do not exist [55], in direct contrast with some experimental evidence. One viable alternative to needle soliton interpretation is to test experimental findings using purely numerical simulations. Even this approach is however extremely difficult because, apart from the complexity of the higher-dimensional system, in analogy to slab solitons and in agreement with experiments, also needle solitons exist when the beam parameters which provide the boundary conditions for the propagation problem are chosen so as to lie as close as possible to the existence curve (which is, of course, a priori, unknown) [54]. More fundamentally, one cannot completely rule out the possibility that some of the approximations introduced in the quasi-analytical slab-soliton model may not be valid in the 2+1D case, or that even the simple Khuhtarev model may not be able to describe in a satisfactory way the physical situation.

Formally, the theoretical "problem" arises when introducing an external bias field, and is strictly connected with the requirement that the space-charge field is to be conservative. In the 2+1D case, the approximated equation relating the space-charge field to the optical field (analogous to eq.(10)) reads

$$\nabla \cdot \left(\boldsymbol{E} \left(I_b + I \right) + \frac{k_b T}{q} \nabla \cdot \left(I_b + I \right) \right) = 0, \tag{13}$$

and its most general solution can be written in the form

$$\boldsymbol{E}(x,y,z) = \frac{\delta\,(1+I(x=\pm\infty)/I_b)}{1+I/I_b}\boldsymbol{e}_x\text{-}\ \tfrac{k_bT}{q}\nabla\cdot\ln\left(1+I/I_b\right)+\tfrac{\delta(1+I(x=\pm\infty)/I_b)}{1+I/I_b}\nabla\times\boldsymbol{v} \quad (14)$$

where $\boldsymbol{e_x}$ is a unit vector in the x direction, δ is associated with the boundary condition of eq.(5), whereas $\boldsymbol{v}(x,y,z)$ is an arbitrary vector field determined by imposing condition of eq.(6). Looking for z independent intensity profiles associated with soliton-like propagation, we can take $\boldsymbol{v}$=(0,0,f(x,y)), where f obeys

$$\nabla_\perp^2 f - \nabla_\perp f\cdot\nabla_\perp\ln\left(1+I/I_b\right) = \frac{k_bT}{q}\frac{\partial}{\partial y}\left(1+I/I_b\right) \quad (15)$$

with $\nabla_\perp=\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y}\right)$ [26].

Self-bending Self-bending and self-deflection are a generic signature of system asymmetry in photorefractive self-trapping [60][61][62][63][64], yet, given the significant advances in the understanding of screening solitons, its study has been mainly associated with steady-state solitons. As mentioned above, in a conventional optical propagation in a photorefractive, in absence of external applied field and photovoltaic effects, spontaneous beam instability leads to beam-fanning. This fanning is a distinct signature of space-charge fields induced by charge diffusion, the only charge separating mechanism in absence of drift, and originates from spurious inhomogeneities in the intensity distribution, due to scattering from crystal imperfections. In the presence of a self-trapped micron-sized beam, however, things are quite different. The marked intensity inhomogeneity of the nondiffracting beam itself engenders a small, but not negligible, symmetric charge distribution that gives rise to an asymmetric field component, and an associated small asymmetric pattern component in the self-lensing structure, along the *entire* propagation trajectory (as opposed to what occurs for a diffracting beam, where self-bending occurs only for the initial stages of propagation). Although self-bending can be neglected in most configurations of interest, it does play a fundamental role in limiting the maximum attainable solitary wave propagation in a photorefractive. This is because self-bending increases nonlinearly along the propagation axis, and, for a long enough propagation, inevitably leads to soliton annihilation. More importantly, soliton annihilation is not a merely geometrical limitation, since it depends, like diffraction, on the transverse spatial scale, being stronger for smaller beams.

3.3 Photovoltaic Solitons

As mentioned earlier some photorefractive noncentrosymmetric crystals, like $LiNbO_3$, $BaTiO_3$, and $LiTaO_3$, manifest the so-called photovoltaic effect [65], that can be generally described by introducing in the Kukhtarev model a photoinduced current component (the last term of eq.(2)) [11]. In most situations, the net effect can be reduced to that of a conventional biased nonphotovoltaic by introducing an effective bias field, the so-called photovoltaic field. This allows, in analogy to screening solitons, photovoltaic solitons[64] [66] [67] [68] . Thus, although the underlying driving mechanism is physically different (giving rise , in some cases, to peculiar phenomenology, as for example in ref.([69])), their description traces the steps indicated above for screening solitons. Experimental proof of bright, dark, and vortex solitons, both in the lower dimensional slab and higher needle cases, has been reported [70] [71] [72].

3.4 Self-trapping in semiconductors

An interesting and potentially useful extension of photorefractive self-trapping phenomenology is represented by the observation of slab and needle photorefractive screening solitons in semiconductor iron doped indium phosphide (InP) [40] [41]. These solitons can be interpreted in much the same manner of their ferroelectric counterparts described above, although they do introduce some major differences. First of all, they can be formed with infrared beams, at typical telecommunications wavelengths. Secondly, they are characterized by shorter response times with respect to their ferroelectric counterparts. Clearly, electro-optic response being weaker than in crystals with spontaneous polarization, self-trapping is obtained for considerably high space-charge fields.

3.5 Self-trapping in paraelectrics

Photorefractives, as a large part of electro-optic crystals, are ferroelectrics. As such, they manifest spontaneous polarization under the critical Curie temperature T_c, passing from their original high temperature centrosymmetric phase to a noncentrosymmetric phase. Although most electro-optic, and consequently photorefractive, experiments are carried out with ferroelectrics in the lower symmetry phase, since the strong spontaneous polarization of poled samples allows considerable responsivity, some research is carried out with samples in the higher symmetry, also referred to as paraelectrics. To enhance the electro-optic response, the crystal is brought in proximity of the phase transition temperature T_c, where the dielectric response is very strong.

Photorefractive screening solitons have been predicted and observed in paraelectric near-transition KLTN [43] [44] [45] , in analogy to screening solitons in noncentrosymmetric photorefractives. In this case, the main difference between the description given above is that the electro-optic response is purely quadratic, and is generally described by the scalar relationship

$$\Delta n = -\frac{1}{2} n^3 \epsilon_0^2 (\epsilon_r - 1)^2 g_{11} E \tag{16}$$

where, in analogy to the noncentrosymmetric case, the electric field E is applied along a given principal axis (in the x direction), the beam is polarized along this axis, and $g_{11}=g_{xxxx}$. Given that KLTN undergoes a structural phase-transition at room temperature, for a range of crystal temperatures T above the transition, ϵ_r takes on values of the order of 10^3 - 10^4. This makes index modulations attainable with reasonably low applied electric fields sufficient to significantly modify beam propagation. The resulting approximate slab-soliton theory is in all similar to the one described for screening slab-solitons, and the final propagation equation reads

$$\frac{d^2u(\xi)}{d\xi^2}=\pm\left(\frac{1}{1+u_0^2}-\left(\frac{1+u_\infty^2}{1+u(\xi)^2}\right)^2\right)u(\xi), \tag{17}$$

leading to spatial bright slab self-trapping when $u_\infty^2=0$ and $u(0)=u_0$, and the minus sign holds, to dark when $u(0)=u_0=0$, and the plus sign holds. In analogy to the noncenytrosymmetric slab-soliton description, we have imposed self-consistently the scalar solitary-wave solution form $A(x,z)=u(x)e^{i\Gamma z}\sqrt{I_b}$, and have normalized the transverse spatial scale to the so-called nonlinear length scale $d=(\pm 2kb)^{-1/2}$, i.e., $\xi=x/d$, with $b=(1/2)kn^2g_{11}\epsilon_0^2(\epsilon_r-1)^2(V/L)^2$, the minus corresponding to bright solitons, whereas the plus holds for dark self-trapping.

As in the noncentrosymmetric case, needle solitons have been documented to be strikingly symmetric, and encounter the same theoretical riddles of their lower symmetry counterparts.

Major differences between photorefractive beam dynamics in ferroelectrics and paraelectrics arise in the mechanisms associated with charge diffusion, a relatively marginal process in noncentrosymmetric trapping [73]. In noncentrosymmetrics, diffusion basically plays the role of an asymmetric seed that eventually leads to appreciable beam bending, and finally to beam annihilation. In a paraelectric, on the other hand, this is not so. The quadratic response makes the index modulation associated with the intrinsically asymmetric diffusion fields (with respect to the beam-intensity symmetry) again symmetric, thus leading to self-lensing in the absence of external bias [74].

Starting from the relationship between $\boldsymbol{E}$ and I, making the approximations indicated in section 3.2, and imposing that the applied voltage be null (in eq.(5)), i.e. $\boldsymbol{J}=0$, gives an approximate expression for the internal field $\boldsymbol{E}=-\frac{k_bT}{q}\frac{\nabla\cdot I}{I_b+I}$, that, inserted into eq.(7) and in eq.(8), gives the final nonlinear propagation equation

$$\left(i\frac{\partial}{\partial Z}+\nabla_\perp^2\right)u+\left(\gamma_1\left(\frac{\partial\left|u\right|^2/\partial X}{\left|u\right|^2+1}\right)^2+\gamma_2\left(\frac{\partial\left|u\right|^2/\partial Y}{\left|u\right|^2+1}\right)^2\right)u=0, \tag{18}$$

where $u=A_xI_b^{-1/2}$, $(X,Y)=2^{1/2}(kx,ky)$, $Z=kz$, $\nabla_\perp^2=\partial^2/\partial X^2+\partial^2/\partial Y^2$ and $\gamma_1=-k^2n^2\epsilon_0^2(\epsilon_r-1)^2g_{11}(K_bT/q)^2$, $\gamma_2=-k^2n^2\epsilon_0^2(\epsilon_r-1)^2g_{12}(K_bT/q)^2$

Neglecting I_b with respect to beam intensity $I(X,Y)$, this equation admits of exact analytical solutions, a remarkable feat in itself given the almost absence of exact solutions in nonlinear propagation problems [75]. These solutions describe a number of interesting optical phenomena, such as self-modified optical diffraction and beam aspect-ratio locking, of which partial experimental observation has been reported [75]. Furthermore, eq.(18) supports a class of Gaussian and non-Gaussian self-trapped solutions in the form of noncircular spatial solitons which, for the currently available values of γ, are outside the reach of observation.

4 Material nonlinearities and solitons

Photorefraction is generally associated with light-matter interaction that does not lead to actual material distortion and can be generally described by a linear relationship between the photoinduced electric field $\boldsymbol{E}$ and the resultant crystal static polarization $\boldsymbol{P}$. Ferroelectrics, in a more general context, respond to local electric fields in a much more complicated fashion, allowing for relevant local reorientation of spontaneous polarization and hence to a series of complex domain phenomena. In standard holographic configurations, this highly nonlinear crystal susceptibility has been implemented for the permanent fixing of a given index patterns. In relation to spatial photorefractive solitons, two different phenomena have been investigated. The first is the fixing of spatial solitons, leading to the permanent imprinting of guiding structures in bulk samples of SBN [76]. A more complicated mechanism can be observed in a paraelectric undergoing a structural phase transition. In this case, light induced diffusion charge fields pin down and seed a self-trapping guiding structure, giving rise to spontaneous self-trapping [77].

5 Nonlinear beam interaction

As is true for a large variety of nonlinear waves in physics, some of the most interesting, counterintuitive, and useful phenomenology is encountered when two or more nonlinear beams are made to interact or collide, and photorefractive self-trapping makes no exception. Actually, in the last decade, photorefractive solitons, in particular screening solitons, have played a leading role in nonlinear collisional studies. This is mainly connected to the fact that photorefractive solitons are supported by a saturated nonlinearity that forwards a more varied phenomenology than more traditional Kerr-like waves. Thus, for example, photorefractive self-trapping occurs both for slab and needle beams in bulk environments, this greatly increasing the degrees of freedom at work. Soliton phenomenology has been investigated in SBN for incoherent screening slab and needle solitons, with the important documentation of beam attraction and ultimately fusion [63] [78] [79] [80]. Phase dependent attraction and repulsion of coherent parallel nonlinear screening needle beams was observed both in BTO [81] and SBN, along with phase-dependent interaction, fusion, and birth [82] [83]. Finally, interaction has been used to investigate symmetry and asymmetry in needle-soliton formation, as mentioned above [57] [69] [59]. More exotic phenomenology directly associated with the higher dimensionality of the system has allowed the documentation of soliton spiraling [46], in which two needle solitons spiral so as to conserve angular momentum, and a collision and interaction of a needle soliton with a slab soliton, in which the different dimensionality of the waves induces intrinsically inhomogeneous interaction forces [84].

Fig. 7. Coherent interaction of a slab and a needle screening soliton in a photorefractive sample of KLTN: (a) input; (b) output in the repulsive case $\Delta\phi_0 = \pi$; (e) output in the attractive case $\Delta\phi_0 = 0$; (c) and (d) is the same as (b) with needle and stripe blocked, respectively; (f) and (g) is the same as (e) with needle and stripe blocked, respectively. Taken from [84]

6 Applications

The intense scientific production associated with photorefractive spatial solitons has to date been mainly directed towards the observation and documentation of the more diverse effects connected to the general traits of soliton physics. Regarding applications, much has been promised regarding optical steering and optical beam handling, but little has been actually done. Pioneering experiments demonstrated the basic but important beam steering capabilities of single photorefractive solitons [24]. The self-induced waveguide formed at a photoactive visible wavelength can be used to guide a longer wavelength nonphotorefractively active beam, such as an infrared signal, through an otherwise bulk environment [85]. Actual beam rerouting is however not feasible, given the time restraints of photorefractive response, that can however be loosened using intense laser pulses [86]. Notwithsatnding this limitation, two conceptual applications that are not directly hampered by slow response have been experimentally demonstrated. The first is a reconfigurable directional coupler obtained simply by launching two independent parallel solitons, and using the double wave-guide structure to observe directional coupler at a longer, infrared, wavelength [49]. The second is connected to enhanced second harmonic generation through self-induced phase-matching of a micron-sized beam throughout a sample of $KNbO_3$, this being a direct consequence of beam self-trapping (similar second-harmonic generation in waveguides) [50]. More recently, direct electro-optic beam handling in paraelectrics through soliton beams has been demonstrated, and promises a less ambitious electro-optic, as opposed to all-optic, beam handling functionality [87].

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