Photorefractive Solitons

E. DelRe[1], M. Segev[2], D. Christodoulides[3], B. Crosignani[1], and G. Salamo[5]

[1] Dipartimento di Fisica, Università dell'Aquila, 67010 L'Aquila, Italy and Istituto Nazionale Fisica della Materia - UdR Roma "La Sapienza", 00185 Roma, Italy eugenio.delre@aquila.infn.it

[2] Physics Department, Technion, Haifa, Israel msegev@tx.technion.ac.il

[3] School of Optics-CREOL, University of Central Florida, Orlando, 32816-2700 United States demetri@creol.ucf.edu

[4] Physics Department, University of Arkansas, Fayetteville, 72703-0000 United States salamo@uark.edu

1 Introduction

Solitons are a universal phenomenon that appears in a wealth of systems in nature. The past few decades have witnessed their identification and observation in the more diverse physical systems: in shallow and deep water waves, charge density waves in plasma, sound waves in liquid helium, matter waves in Bose-Einstein condensates, excitations on DNA chains, "branes" at the end of open strings in superstring theory, domain walls in supergravity, and many more. Solitons can even appear for electromagnetic waves in *vacuum*, where they are supported by QED nonlinearities. And, of course, in Optics: here solitons truly manifest themselves in a large variety of settings (for a review on optical solitons see Refs.[1, 2, 3, 4]). In all these diverse systems, that vary in almost everything from size to dimensionality, from underlying forces to physical mechanisms, propagation leads to a family of nonlinear waves - solitons - that have the same, universal, features: they are all self-trapped entities possessing particle-like behavior.

In this Chapter we outline the mechanisms through which photorefraction can support optical spatial solitons (for a specific review, see pages 61-125 in [4]), give an account of the development of the main underlying ideas, and describe the associated phenomenology. Since the discovery of photorefractive solitons in 1992 [5], they have become one of the most important experimental means to study universal soliton features. The rich diversity of photorefractive effects has allowed experimental investigations into a large variety of soliton phenomena, many of which have here found their very *first* observation, in any soliton-supporting system in nature. For example, it was with photorefractive solitons that fascinating soliton interaction effects, such as 3D soliton spiraling, fission and annihilation, were first demonstrated. Likewise, random-phase (or incoherent) solitons were first observed in photorefractives, and so multi-mode solitons, both in 1D and in 2D. And, very recently, solitons in two-dimensional nonlinear photonic lattices were demonstrated, once again, in photorefractives, using a real-time optically-induced photonic lattice. Today, almost 12 years after their discovery, photorefractive solitons have evolved into a major subject contributing to research at the cutting edge of nonlinear science. The intrinsic complexity of the photorefractive light-induced index-change, driven by several charge transport mechanisms, utilizing linear and quadratic electro-optic effects, and having a polarization-dependent tensorial behavior, contributes in giving rise to a rich phenomenology. Much is understood now about the formation processes of the various types of photorefractive solitons, and many of the parameters can be controlled individually: at the same time numerous questions are still open. Also important, research on solitons in photorefractives has benefited not only the soliton community, but has also introduced a number of new ingredients to photorefractive studies at large. For example, understanding the propagation dynamics of beams in photorefractives (rather than two- and four-wave-mixing), including the formation of self-oscillators (e.g., the so-called "double phase conjugator") has considerably benefited from the understanding gained in photorefrative soliton research. Likewise, exploring spatially-localized effects that emerge and find their full realization directly *within* the sample, such as instabilities and spontaneous pattern formation, with and without a cavity, is now nicely understood through the intimate connection between solitons and modulation instability [4]. These distinguish soliton phenomenology from holographic and wave-mixing schemes, which, by nature, deal principally with the manipulation of light as a device to an effect which occurs *outside* the crystal. In this Chapter, we attempt to provide an updated overview on the fascinating phenomenon of photorefractive solitons, which continuously brings in new surprises, new effects, new excitement, to the photorefractives community, to the much broader nonlinear optics community, and to soliton science at large.

2 The discovery of solitons in photorefractives

In the wake of renewed interest in soliton propagation, triggered by studies on temporal solitons in an ever more competitive fiber optic network, the beginning of the 1990s see an intense effort aimed at finding accessible physical systems in which to experimentally investigate spatial solitons. Conventional Kerr-type nonlinear schemes presented crippling limitations connected to the extremely high optical intensities involved, and suffering from the fundamental constraints associated with the instabilities and catastrophic collapse of Kerr solitons in bulk media. Motivated by the strong nonlinear response of photorefractive crystals, even at low optical intensities, M. Segev, B. Crosignani, and A. Yariv, were able to formulate in 1992 the first photorefraction-based self-trapping mechanism. This embryonic idea sets the beginning of the field, and, indeed, of our description [5].

A spatial soliton is a beam which, by virtue of a robust balance between diffraction and nonlinearity, does not change its shape during propagation. A direct observation of a spatial soliton in photorefractives is shown in Fig.(1).

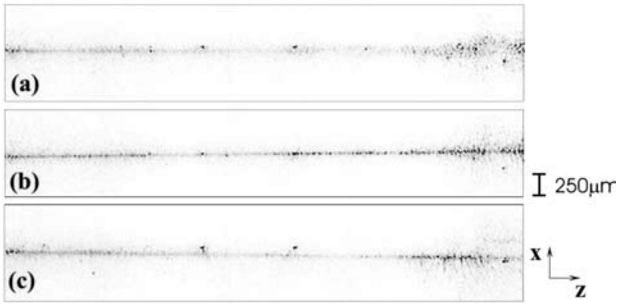

Fig. 1. Top view of spurious scattered light produced by transient centrosymmetric solitons in a biased sample of potassium-lithium-tantalate-niobate (KLTN). (a) Linear diffraction from an input Full-Width-Half-Maximum (FWHM) of 6 μm to 150 μm. (b-c) Solitons formed with opposite values of external field. Taken from [11]

Before the discovery of photorefractive solitons, nonlinear optics in photorefractives was centered on diffusion-driven wave-mixing schemes, typically resulting in high energy exchange between the interacting waves, a mechanism at the heart of photorefractive self-oscillation and "passive" phase conjugators. During that phase, other settings, which exhibited elevated phase-coupling (and much lower energy-exchange) attracted only remote interest. Furthermore, during diffusion-driven photorefractive wave-mixing, beam energy is spontaneously driven into modes not present in the launch, leading to the highly deteriorating phenomenon called beam fanning: the exact *opposite* of a precursor to the orderly events which accompany a solitary wave. Evidently, at the time there was no indication of any kind that photorefractives could support solitons. Segev et al., however, argued that since wave-mixing was intrinsically accompanied by a mutual phase-modulation, one could find a condition in which the plane-wave components of a diffracting beam could mix so as to lead to a trapping self-phase modulation: their mutual exchange could compensate for the linear dephasing between the plane-wave components of a beam, and thus counteract diffraction altogether. They observed that, in contrast to schemes where the main mixing agent is diffusion, leading to the highly asymmetric fanning process, a symmetric mutual phase-modulation could be achieved through the application of an *external bias field*. They concluded that, when the diffusion space-charge field could be neglected with respect the external bias field, a symmetric self-focusing occurs and self-trapping effects should emerge. Under such conditions, fanning itself, being mediated by diffusion, would have a secondary effect [6].

In order to achieve a set-up symmetric with respect to the propagating axis, the beam should be launched in a zero-cut uniaxial sample along the ordinary axis a, with the external biasing field E_0 applied along the poling optical axis c, through two electrodes brought to a relative potential V.

An optimal arrangement was identified in Stontium-Barium-Niobate (SBN) doped with rhodium impurities. Characterized by an $r_{33} \simeq 220pm/V$, for an extraordinary c polarized beam launch, index modulations of the order of $|\Delta n| \simeq (1/2)n^3 r_{33} E_0 \sim 2 - 5 \cdot 10^{-4}$ for achievable fields of the order of $E_0 \sim$ 1-3kV/cm could be reached, a nonlinearity sufficient to support the formation of a 10 μm wide soliton.

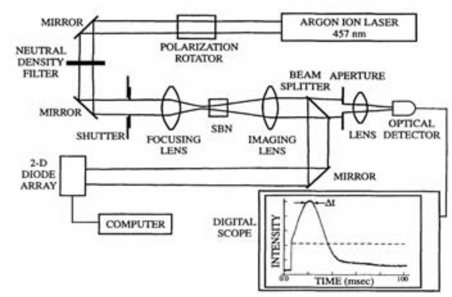

Fig. 2. The original soliton observation schematic, as reported in [7]. The extraordinary launch beam is focused in proximity of the input facet. Transient dynamics were detected recording the output intensity transmitted through an appropriate aperture positioned before the output detector.

The first pioneering experiments, reported in [7] were carried out with the set-up indicated in Fig.(2). In relating the first results, we more often delve upon details of transient - as opposed to steady-state - effects, and miss the main and revolutionary point: where no previous phenomenology even hinted at self-focusing, these first experiments indicated, unmistakably, that a visible continuous-wave beam propagating in a biased sample would actually self-trap, the ensuing spatial soliton being readily accessible to direct observation (see Fig.(1)). Although the initial conjecture would later not prove to be greatly adherent with first findings, it did express the main physical idea supporting the phenomena. These pioneering results established that very narrow beams launched in a properly-biased photorefractive crystal would self-trap, and propagate in a robust fashion, undistorted by fanning and other noise sources in the crystal or even fairly large deviations from optimal launch conditions [8]. For example, it was established that a 15 μm sized continuous-wave 457nm μW beam would not suffer fanning and self-trap for external fields from 400-500 V/cm. This observation led to a rapid series of predictions and experiments, which now form the phenomenological basis of photorefractive spatial solitons [8, 9, 10].

As commonly occurs when scientific progress is in action, those first experiments posed more questions than answers. First, the results indicated a *transient* self-trapping process (see insert in Fig.(2)), during a time window of several hundreds of milliseconds, *not* characterized by stringent existence conditions, as would have been expected for normal self-trapping, where diffraction is exactly balanced by a specific value of nonlinearity. On the contrary, beam intensity and applied bias field, two parameters assuredly implicated in the nonlinear response, could be considerably varied without significant changes in self-trapping. But even more astonishingly, the tentative launch of two-dimensional (circular) beam led to its stable self-trapping. This at once indicated that the underlying nonlinear mechanism was not Kerr-like, since the catastrophic collapse associated with self-focusing in Kerr media did not occur. For some reason, the intrinsic anisotropy of the light-induced space-charge-field, resulting both from the application of an external bias along one transverse direction only, and the directionally resolved electro-optic response, allowed for a two-dimensional soliton [7, 8, 9]. Interestingly, both aspects, the transient/non-parametric and quasi-circularly-symmetric nature of these first manifestations, which are reproducible in all photorefractive materials that can support solitons, are still not fully understood even today (see sections 4 and 5). The observation of a two-dimensional soliton in a bulk medium has attracted interest and sometimes fomented outright debate. At the same time, the transient nature was generally looked upon as an undesired and limiting effect. It cast a shadow both as to the nature of the interaction, but more importantly, as to its stability. Transient effects of the sort had a history in photorefraction, and they were attributed to charge accumulation in dark regions of the sample of the photo-excited charge, depleting illuminated portions and possibly screening external bias.

This triggered the idea that the transient nature of the self-trapping was an effect of charge accumulation screening E_0: free-charges would be photoexcited across the beam profile, and, drifting in the external field, would reach the bordering dark regions, and get trapped there. These trapped charges would give rise to an internal (space-charge) field with a polarization opposite to E_0. In SBN, this lowering (screening) of the applied field at the illuminated regions would locally lead to electro-optic lensing. The decay of this (induced) lens with time would then be a consequence of the fact that charge would continue to separate until E_0 was totally screened, thereby saturating and flattening the induced ”lens”. In this, investigators found the solution: they would foresee a compensating mechanism through which accumulated charge could be eliminated by homogeneously illuminating the sample, which amounts to increasing the dark sample conductivity (see Fig.(3)). On the basis of the relative intensity of the beam to the background, there would exist a dynamic equilibrium leading to a steady-state lensing effect [12, 13, 14, 15].

As the model was reformulated on this new, an to some extent, simpler screening idea, foreseeing the artificial enhancement of crystal dark conductivity, Castillo et al. [16] reported steady-state self-focusing, using a homogeneous illumination to free accumulated charge. Finally, Shih et al. [17, 18] and Kos et al. [19] were able to not only observe this non-transient soliton phenomenology, but also relate it to the new model. This type of self-trapping is

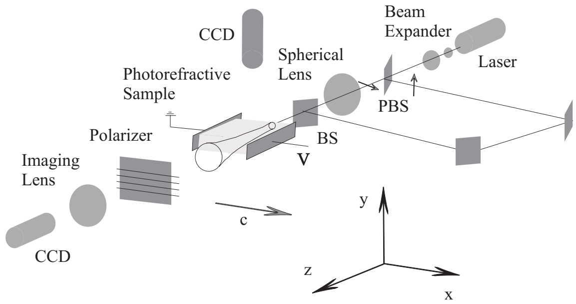

Fig. 3. Scheme used to stabilize screening solitons, as described in [17] and [19]. Now the diffracting launch beam is accompanied by an ordinary unfocused wave appropriately extracted from the single laser.

since termed a *screening soliton*, and constitutes the most commonly studied type of photorefractive soliton. Its explanation requires a treatment which goes beyond the small modulation depth treatment, and for which single mode description, such as the involved in two-wave mixing, is unsatisfactory.

Photorefractive solitons have since been observed in SBN, BSO, BGO, BTO, $BaTiO_3$, $LiNbO_3$, InP, CdZnTe, KLTN, $KNbO_3$, polymers and organic glass.

3 A saturable nonlinearity

The formulation of a descriptive and predictive theory for photorefractive solitons involves some profoundly different aspects and theoretical tools than those employed in traditional wave-mixing theories. First, no periodic structure is present, and second, in most configurations, all the physical variables vary across the beam profile by a large fraction (e.g., from peak to zero intensity) such that the modulation cannot be treated as a small perturbation. However, steady-state photorefractive solitons have two intrinsic symmetries which reduce the problem: they are evidently time-independent, and their intensity I is independent of the propagation coordinate z. Yet the heart of complexity is nonlinearity, and even for a z-invariant photoionizing intensity I there is still a wide range of parameters, of which only a small subset can support solitons. In order to formulate a semi-analytic theory, a one-dimensional reduction must be implemented: the beam should be such that no y-dynamics emerge, the soliton intensity being solely x-dependent $[I(x)]$. Experimentally, this was achieved by launching a beam focused down through a cylindrical lens, and quite similarly this has led to quasi-steady-state self-trapping in the absence of background [9], and to steady-state screening solitons for appropriate values of E_0 (see Fig.(4)) [19].
In fact, in many aspects these one-dimensional (stripe) waves, generally termed one-plus-one dimensional screening solitons [(1+1D)], share with

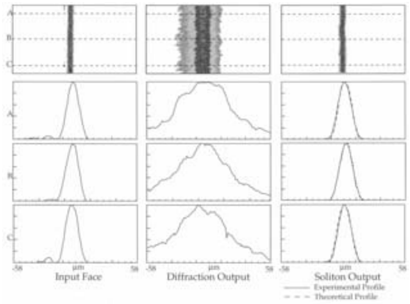

Fig. 4. A one-dimensional soliton observed in biased SBN. Top: input intensity distribution, output linear diffraction (no bias), and output self-trapped beam. Center and bottom: profiles. As reported in [19]

their needle-like counterparts, which are therefore two-plus-one-dimensional screening solitons [(2+1D)], the same behavior. Although (2+1)D self-trapping in photorefractive media is still not fully theoretically understood (see section 4), the theory of (1+1)D photorefractive solitons [12, 13, 14, 15] constitutes the basic confirmation that photorefraction supports self-trapping.

In most conditions of interest, the optical intensity distribution $I(x)$ is such that, for an electron-dominated photorefraction, the resulting concentration of photo-excited electrons N, the concentration of acceptor impurities N_a, and the concentration of donor impurities N_d follow the hierarchy $N << N_a << N_d$. Under these generally valid assumptions, the space-charge field E is related to the optical intensity I through the nonlinear differential equation [20]

$$\boldsymbol{E}(I_b + I)\frac{1}{1+\frac{\epsilon}{N_a q}\frac{d\boldsymbol{E}}{dx}} + \frac{k_b T}{q}\frac{d}{dx}\left((I_b + I)\frac{1}{1+\frac{\epsilon}{N_a q}\frac{d\boldsymbol{E}}{dx}}\right) = g. \qquad (1)$$

Here, ϵ is the sample dielectric constant, q is the electron charge, k_b is the Boltzmann constant, T is the temperature, and g is a constant related to the boundary conditions, i.e., to the voltage V applied on the x-facets L_x apart. I_b is the so called background illumination, the homogeneous optical intensity that allows a finite crystal conductivity. We emphasize that none of the small-modulation methods can be used to linearize Eq.(1) and solve for solitons. Instead, a profoundly different approach has to be taken.

3.1 Nonlinearity

The structure of Eq.(1) and the presence of the term $(1+\frac{\epsilon}{N_a q}\frac{d\boldsymbol{E}}{dx})$ indicate a natural normalization procedure to enact an approximate approach: $Y = E/E_0$, $Q = (I_b + I)/I_b$, and $\xi = x/x_q = x/[\epsilon E_0/(N_a q)]$. We might note that, using the conventional scaling implemented for wave-mixing, x_q can be identified with a saturation scale, i.e., that spatial scale under which the maximum attainable charge (when the concentration of ionized donors $N_d^+ \approx N_a$) cannot screen E_0 (which becomes comparable with the saturation field E_q). From Eq.(1) Y and Q are now related through

$$\frac{YQ}{1+Y'} + a\left[\frac{Q'}{1+Y'} - \frac{Q}{(1+Y')^2}Y''\right] = G, \tag{2}$$

with $a = N_a k_b T/\epsilon E_0^2$ and $G = gE_0/I_b$. The prime stands for $(d/d\xi)$. Equation (2) can be formally solved, without approximations, for Y to give

$$Y = -a\frac{Q'}{Q} + \frac{G}{Q} + \frac{GY'}{Q} + a\frac{Y''}{1+Y'}. \tag{3}$$

We can now identify the various terms with precise physical processes, as we shall see. Equation (3) is rendered tractable by the fact that the greater part of spatial soliton studies involve the trapping of beams with an intensity Full-Width-Half-Maximum (FWHM) $\Delta x \sim 10\mu m$. For most configurations, $x_q \sim 0.1\ \mu m$, and $\eta = x_q/\Delta x \sim 0.01$ represents a smallness parameter. In other words, screening solitons do not deplete photorefractive charge. A dimensional evaluation of the various terms for the appropriate high-modulation regime indicates that

$$Y^{(0)} = \frac{G}{Q} + o(\eta), \tag{4}$$

since $a \sim 2.5$, and $G \simeq$-1 [15]. A first correction is obtained by iterating this solution into Eq.(3), and the resulting expression for Y is

$$Y^{(1)} = \frac{G}{Q} - a\frac{Q'}{Q} - \frac{Q'}{Q}\left(\frac{G}{Q}\right)^2 + o(\eta^2). \tag{5}$$

The first dominant term is generally referred as the screening term. It represents, in our discussion, the main agent leading to solitons. It is local, in that it does not involve spatial derivatives or integration, has the same symmetry of the optical intensity Q, and represents a decrease in E with respect to E_0 on consequence of charge rearrangement ($G \simeq$-1). So perhaps the most astonishing fact of our discussion is that, in truth, for a large variety of conditions, this form of self-focusing (including self-defocusing) *is* the dominant effect, as the plentiful family of reported observations that have followed the 1992-1993 discovery imply. The second term, of first order in η, is simply the high-modulation version of what is generally called the diffusion field (E_d). The third, again of first order in η, is the coupling of the diffusion field with the screening field, a component sometimes referred to as deriving from charge-displacement [21]. Both these two last terms are nonlocal, in that they involve a spatial derivative, and thus provide an anti-symmetric contribution to the space charge field (Y) for a symmetric beam $I(x) = I(-x)$. That is, these last two terms lead to a beam self-action of opposite symmetry of that required to support solitons: whereas for conventional mixing studies they play the central role, here they lead to beam self-bending (or self-deflection), which, for most configurations, amounts to a slight parabolic distortion of the preferentially z-oriented trajectory. The subject has attracted interest over the years and has helped build an understanding into the limits of the local saturable nonlinearity model [10, 18, 21, 22, 23, 24, 25, 26, 27, 28, 29].

In order to identify the nonlinearity, we must now translate the space-charge field E into an index modulation. The standard soliton set-up is such that a negative uniaxial zero-cut sample is positioned so that the x-axis is the direction along which E_0 is applied, the soliton beam of intensity I is extraordinarily-polarized and is propagating along z, while I_b is obtained through a co-propagating ordinarily-polarized plane-wave [17]. For a non-centrosymmetric photorefractive, like SBN, $\Delta n = -\frac{1}{2}n^3 r_{33} E$, n being the unperturbed crystal index of refraction, and r_{ij} the linear electro-optic tensor of the sample. On taking, consistently with our iterative scheme, the expression of Eq.(4), we obtain the nonlinearity

$$\Delta n(I) = -\frac{1}{2}n^3 r_{33}\frac{V}{L_x}\frac{1}{1+I/I_b} = -\Delta n_0 \frac{1}{1+I/I_b}, \tag{6}$$

which constitutes a saturable nonlinearity, identical (within a constant term) to the nonlinear index change in a homogeneously-broadened two-level-system.

3.2 The soliton-supporting nonlinear equation

A soliton is loosely defined as a wave that preserves its shape and velocity throughout propagation, while, very importantly, displaying a particle-like behavior when made to interact ("collide") with other solitons. As such, solitons possess a number of conserved quantities, such as power, momentum, Hamiltonian, etc. [4]. Optical spatial solitons, in their scalar manifestation, are generally governed by the nonlinear equation for a monochromatic paraxial beam

$$\left[\frac{\partial}{\partial z} - \frac{i}{2k}\frac{d^2}{dx^2}\right] A(x,z) = -\frac{ik}{n}\Delta n A(x,z) \tag{7}$$

where $k = 2\pi n/\lambda$ is the wave-vector, A is the extraordinary component of the slowly varying optical field, i.e. $E_{opt}(x,z,t) = A(x,z)exp(ikz - i\omega t)$, $\omega = 2\pi c/n\lambda$, and $I = |A(x,z)|^2$. We seek stationary (non-diffracting) solutions of the form $A(x,z) = u(x)e^{i\Gamma z}\sqrt{I_b}$, normalize the transverse spatial scale to the so-called nonlinear length scale $d = (\pm 2kb)^{-1/2}$, i.e., $\xi = x/d$, which for photorefractive solitons is obtained from the expression $b = (1/2)kn^2 r_{33}(V/L_x)$, and obtain [12, 15]

$$\frac{d^2 u(\xi)}{d\xi^2} = \pm\left(\frac{\Gamma}{b} - \frac{1}{1+u(\xi)^2}\right)u(\xi). \tag{8}$$

The plus sign is for $b > 0$, the minus for $b < 0$. The sign of b corresponds to the sign of Δn_0, and implies a self-focusing, for $b > 0$, or a self-defocusing, for $b < 0$, nonlinearity, having established that E decreases across the beam profile. Applying the external bias in a particular direction with respect to the crystalline axes univocally establishes the sign of the nonlinearity, through the sign of r_{33}. For example, in SBN applying E_0 in the direction of the crystalline (ferroelectric) c axis implies $b > 0$, and we observe a self-focusing nonlinearity. It is possible to apply E_0 in a direction opposite to ferroelectric axis, thus effectively changing the sign of b, then E_0 must be smaller than the coercive field, otherwise it may render the ferroelectric crystalline structure unstable and de-pole the crystal. Both defocusing and focusing nonlinearities support solitons. A self-focusing nonlinearity traps a conventional bell-shaped beam into a bright soliton; a self-defocusing nonlinearity traps a wave feature, such as a notch generated by a π phase jump, whose structure is enlarged by the diffraction of surrounding illuminated regions, the ensuing propagation invariant wave being termed a dark soliton.

Equation (8) can be integrated (by quadrature) once, giving the relationship $\Gamma/b = \log\left(1+u_0^2\right)/u_0^2$ for bright beams, and $\Gamma/b = 1/\left(1+u_\infty^2\right)$ for dark, where $u_\infty = u(\infty) = -u(-\infty)$, and $u_0 = u(0)$ ($u_0^2 = I(0)/I_b$ being referred to as the intensity ratio).

3.3 Soliton waveforms and existence curve

As can be imagined, the self-trapped waves u, solutions of Eq.(8), form an isolated subset of all possible dynamical evolutions described by Eq.(7). The imposition of z-invariance implies not only a specific relationship between beam parameters, but fixes the actual waveform u in all its details (see Fig.(5)). In general, such a prediction can have little if no physical meaning. For solitons, however, we have two countering effects, diffraction and self-focusing, which are coupled by nonlinearity to form a feedback mechanism. The result is that most soliton solutions are stable to perturbations, and represent an attractor to system dynamics. This, in turn, contains the beauty and physical appeal of soliton physics: that a propagating beam, interacting in a non-trivial way with a hosting medium, should preferably be attracted to a robust, propagation-invariant, and very specific wave-form, instead of producing an unstable and unpredictable chaotic system.

Returning to our system, what are the soliton wave-forms, and, more importantly, what are the beam parameters, $\Delta\xi$ (associated to Δx) and intensity ratio u_0^2, that characterize the subset of soliton solutions? The issue is of particular importance, because, although self-trapping forms an attractor, launch experiments are designed to deterministically lead to a soliton, a scheme that requires the launch to be close enough to a self-trapped wave in parameter space (intensity u_0^2 and normalized width $\Delta\xi$). For the family of integrable nonlinear equations, such as the Sine-Gordon, the Nonlinear Schroedinger, and the Korteweg and de-Vries equations, an explicit solution can be found, for which the relationship between beam $\Delta\xi$ and u_0^2 is unique. However, for the saturable nonlinearity described by Eqs.(6,8) the $\Delta\xi$ versus u_0^2 relation is not unique, but instead yields a continuous curve commonly termed the soliton existence curve [15] (see Fig. (6)).

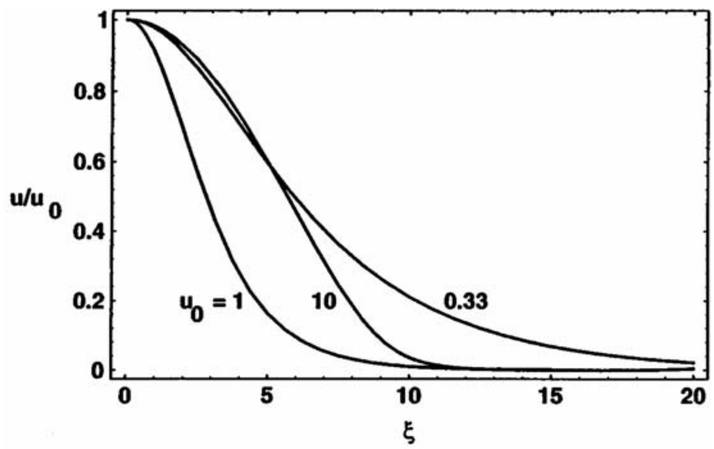

Fig. 5. Soliton waveforms for bright solitons, as reported in [19].

The wavefunctions of bright photorefractive screening solitons are bell-shaped functions which are neither a Gaussian nor a hyperbolic secant [30]. Experimentally, such solitons are generated by launching a focused-down one-dimensional Gaussian beam with a Δx and peak intensity I_0 such that, for the given configuration of crystal parameters n, r_{33}, and I_b, for the given bias E_0, the resulting values of $\Delta\xi$ and u_0 lie on the existence curve. For dark solitons, in turn, the same procedure can be implemented for the relevant (u_∞,$\Delta\xi$) parameter space.

3.4 Experiments and theory

The main advantage of having formulated the theory highlighting the saturable nature of the nonlinearity is that, within the limits in which the underlying approximations are valid, it allows the prediction of solitons as a specific feature independent on the particular experimental configuration. Thus a physically *identical* soliton will emerge for two self-trapped beams of, say 10 and 20 μm, as long as the applied voltage V, material response,

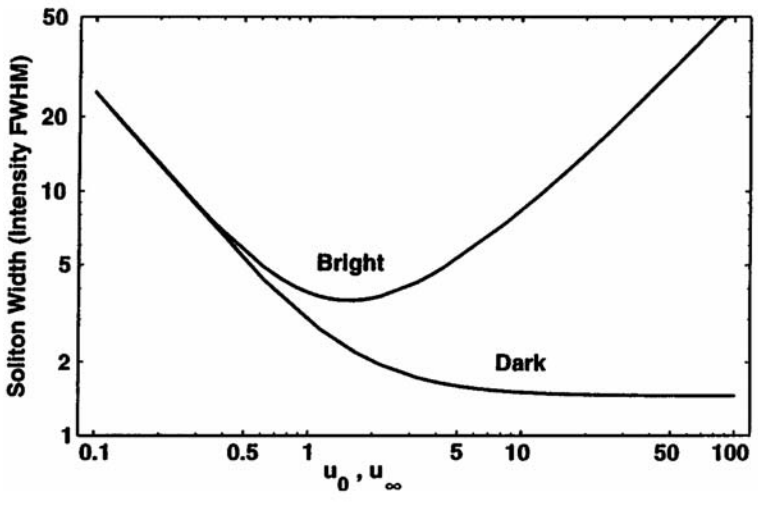

Fig. 6. Soliton bright and dark existence curve, as reported in [19].

beam intensity and background illumination are such as to project the two conditions on the very same point on the (u_0, $\Delta\xi$) parameter space. Note that this powerful device breaks down as soon as we consider the o(η) terms, or even simple corrections in the actual nonlinearity, such as those deriving from dielectric nonlinearity [31].
In order to test the validity of the treatment, experiments have been carried out with the aim of detecting for what conditions of launch a 1+1D steady-state soliton would form [19, 32, 33], and some examples of results are contained in Fig.(7) for bright solitons and Fig.(8) for dark.

Indeed other experiments have led to much the same conclusions, for which the qualitative agreement is full, but for some regions of parameters, where the quantitative comparison is weaker. At present it is believed that the discrepancy is due more to the presence of extraneous effects than to a deficiency in the model. For example, it has been observed that some of the background illumination, which is made to propagate along an ordinary mode, in order to not undergo substantial evolution, is actually trapped by the soliton through the finite r_{13} coefficient. Another source of uncertainty is connected to the difficulty in establishing the precise value of r_{ij} for the given sample: this can depend on the level of purity of the poling, on the presence of considerable clamping and on temperature.

Concerning more fundamental aspects of the model, we note that although the results shown in Fig.(7) support the approximation contained in Eq.(4), the actual beam evolution shows a clear and reproducible self-bending effect

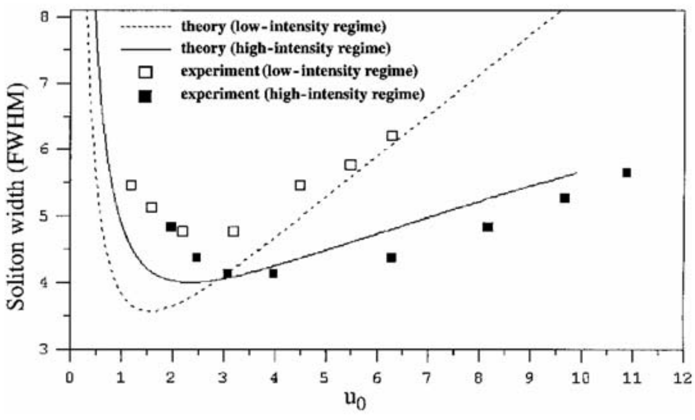

Fig. 7. Comparison between experiments and theory for (1+1)D bright screening solitons, from [32]. Here the low-intensity regime is what we specifically term screening solitons.

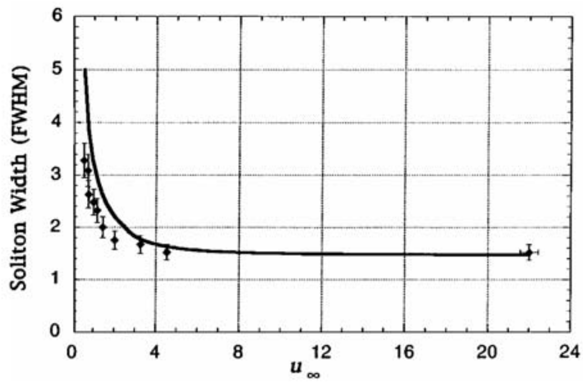

Fig. 8. Comparison between experiments and theory for (1+1)D dark screening solitons, from [**?**]

that can only find its explanation in the full expression of Eq.(5). It appears that, even though these nonlocal terms produce a parabolic trajectory, they do not greatly influence the existence curve. From a different perspective, we note that self-bending becomes an important issue when experiments are carried in highly solitonic regimes, characterized by a large ratio between self-trapped propagation distance and linear diffraction length L_z/L_d: to increase L_z/L_d we can either use a longer sample, this increasing nonlinearly the lateral shift, or use tighter launch beams, this increasing η.

4 Two-dimensional solitons

The (1+1)D screening solitons represent the firm experimental and theoretical foothold on which a large part of research rests, especially because of the appealing nature of the saturable nonlinearity which gives rise to numerous (fascinating) soliton interactions not present with Kerr-type solitons. However, the most ground-breaking achievement, both from the physical and from the applicative point of view, is the self-trapping of (2+1)D solitons. As their lower-dimensional counterparts, needles form both in the transient regime - as quasi-steady-state self-trapping [7] - and in temporal steady state as (2+1)D screening solitons [17, 18]. Such needle-like solitons were originally documented in SBN, and have been replicated in most soliton-supporting photorefractive media, such as other ferroelectrics [34], semiconductors [35], paraelectrics [36], sillenites [37], and indeed for most types of self-trapping: bright, photovoltaic [38], multimode [39, 40, 41], and incoherent [42, 43], to name a few. Furthermore, even (2+1)D dark solitons were observed in photorefractives, in quasi-steady-state [44] and in steady-state [45] under a bias field, as well as photovoltaic [46] and incoherent [47] dark "vortex" solitons.

An example of a dark vortex screening soliton is shown in Fig.(12). Two-plus-one dimensional solitons form when a circularly symmetric beam is focused down onto the input face of the sample, and the initially diffracting beam collapses into a non-diffracting 2D beam having an almost ideally circularly-symmetric shape (see Fig.(9)). These studies, which constitute one of the rare possibilities of observing (2+1)D solitons, have greatly contributed to the understanding of the physics associated with higher-than-one-dimensional nonlinear waves. The observation of soliton spiraling in full 3D [48], as well as fusion, fission, birth and annihilation of (2+1)D solitons [49, 50, 51], have extended the very *concept* of soliton-particle behavior.

The description and understanding of the mechanisms that support (2+1)D solitons are, to some extent even today, still incomplete. This is because the propagation involves a nontrivial three-dimensional, anisotropic, and spatially nonlocal nonlinear problem [52, 53, 54, 55, 56, 57]. The fact that photorefractives can support self-trapped needles of (almost ideally) circularly-symmetric shape seems very surprising right from the outset. More

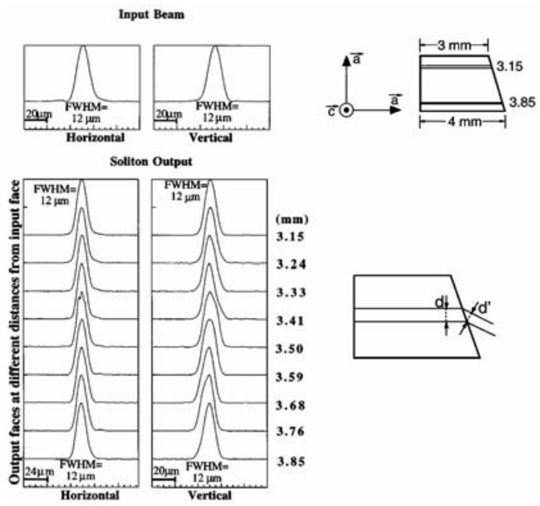

Fig. 9. The direct and detailed observation of a circularly-symmetric 12 μm needle phenomenology in a non-zero-cut sample of SBN, from [18].

specifically, the external field is applied between two parallel planar electrodes, and thus breaks the circular symmetry of the problem. The explanation relates to the fact that the (space charge) field lines bend in the regions of higher illumination, and, for some range of parameters seem to yield a quasi-radial distribution of the field component giving rise to a nonlinear index change. For example, in SBN this means that the c-component of the space charge field has roughly a circular symmetry. The very fact that such (2+1)D propagate in a stable fashion, not undergoing catastrophic collapse (as such solitons in Kerr media would), is a direct indicative that the photorefractive nonlinearity is saturable also in two (transverse) dimensions. However, during the temporal transients, and for various values of applied field, self-trapping manifests considerable beam ellipticity [52], excluding the direct validity of a circularly-symmetric 2D saturable model. Nevertheless, the large amount of experimental evidence on 2D solitons in almost every photorefractive material in which solitons have been identified, implies that a modified model, possibly anisotropic and slightly nonlocal, should exist [55]. What clears the picture are the studies on (2+1)D soliton interaction-collisions. On the one hand, spiraling and large angle collisions show that whatever anisotropic components emerge, they do remain localized around the beam [48, 58], whereas lower angle collisions indicate unmistakably the presence of a saturable yet anisotropic nonlinear behavior: the repulsion of incoherent needles [59].

From a theoretical perspective, the system has two fundamental anisotropies: the boundary conditions, which emerge from the application of an external bias along the x-direction, and the electro-optic response, which implies a complex tensorial index modulation depending on the beam polarization, direction of the local electric field $\mathbf{E}$, and the relative orientation with respect to the crystal lattice. The result is that the nonlinear response has a nonlocal component that is superimposed on the saturable component [54]. In order for a quasi-circular optical symmetry to appear, the underlying space-charge field $\mathbf{E}$ must be anisotropic, manifesting two characteristic lateral lobes (see Fig.(10)) [60]. The appearance of these features, that have an increased complexity with respect to the ionizing intensity $I(x, y)$, are the basic manifestation of a nonlocal mechanism. Contrary to what might seem logical, if one attempts to identify their origin extending our knowledge of (1+1)D solitons, these lobes do *not* emerge because of the nonlocal mechanisms indicated in Eq.(5). These lobes emerge due to the matching of boundary conditions in the higher dimensional case, i.e., they match the local field structure, in the beam cross-section, to the x-oriented $\mathbf{E} = \mathbf{E}_0$ far from the beam.

In the 2+1D case we start from the basic relation, valid to zero order in η,

$$\nabla \cdot (\boldsymbol{Y} Q) = 0 \tag{9}$$

and the irrotational condition

$$\nabla \times \boldsymbol{Y} = 0. \tag{10}$$

From these a lobular structure emerges (see Fig.(11)). This anisotropy which is not at all connected to η, is simply not small in *any* accessible case. In other words, the nonlocality in the (2+1)D case is intrinsic to the process, whereas it is merely a correction for (1+1)D solitons.

The fact that 2+1D solitons are supported by this more complex nonlinearity does not substantially modify our soliton picture. One burdening consequence, however, is that we do not have a means to formulate in a straightforward manner an existence curve for (2+1)D solitons, nor can we unequivocally say if such a concise and powerful tool even exists for the higher dimensional solitons. Nevertheless, if we phenomenologically build the set of points in which it is possible to observe circular-symmetric self-trapping [18, 36], we find a single valued continuous curve that behaves and looks just like the existence curve of (1+1)D solitons (albeit at somewhat higher values of $\Delta\xi$).

Evidence on both the existence of circularly-symmetric solitons and of their intrinsic difference from (1+1)D solitons in photorefractives is highlighted by the experimental studies on transverse instability of (1+1)D solitons in bulk media. In those experiments, increasing the nonlinearity leads a (1+1)D soliton (at the proper value of nonlinearity vs intensity ratio, as

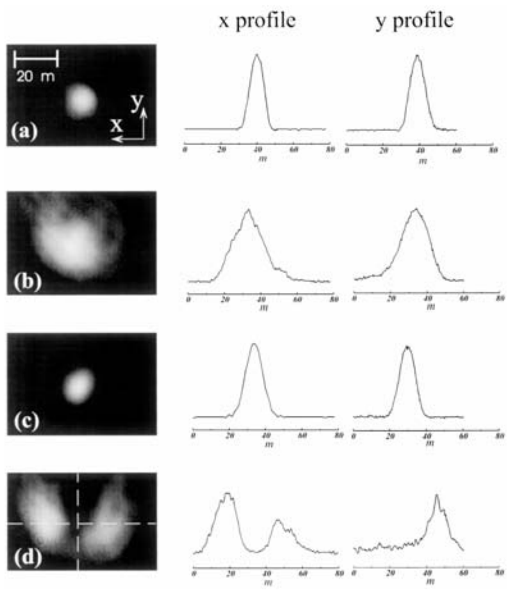

Fig. 10. Needle electro-holography from [60]. (a-c) conventional formation of a centrosymmetric soliton; (d) electroholography showing the lobes.

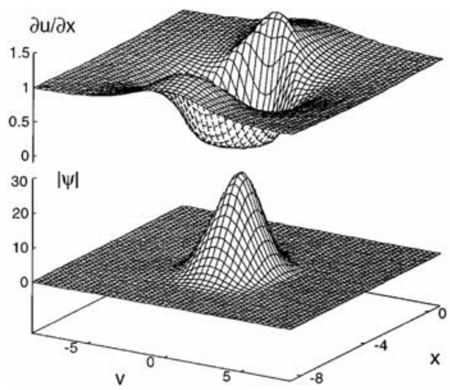

Fig. 11. Numerically evaluated x-component of E (top) for the soliton beam profile (bottom), from [55].

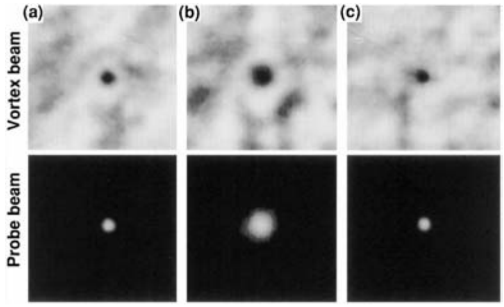

Fig. 12. A screening vortex and its guiding features, from [45]. (a) Input intensity distribution of the vortex; (b) Diffracting vortex after linear propagation to the output of the sample; (c) Self-trapped output intensity distribution in a biased sample. (bottom) Probe beam guided propagation.

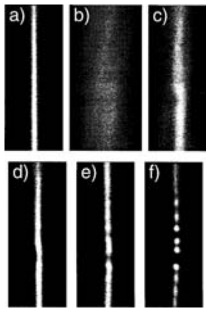

Fig. 13. Transition from (1+1)D to (2+1)D self-trapping, from [61].

determined by the existence curve); then, a further increase in the nonlinearity results in beam breakup into an array of *circularly-symmetric* solitons, as shown in Fig.(13) [61].

5 Temporal effects and quasi-steady-state dynamics

One of the main characteristics of photorefraction is that it is a cumulative process. This is the origin of its elevated response even at low optical intensities. Evolution is dominated by a charge redistribution process which leads to a time constant $\tau \propto 1/I$. For small-modulation depth conditions I is almost constant, and a single-time-constant behavior can be identified. For spatial solitons the story is quite different: not only are we faced with a high modulation response, and the time constant changes in the transverse plane, but since they form out of a three-dimensionally resolved diffracting beam, time behavior is also resolved in the propagation direction. The result is an evolution which presents a number of surprising phenomenologies, which are, however, complicated and difficult to describe (an example that goes beyond soliton studies can be found in [62]).

Possibly the most important, and, indeed, surprisingly, effect is observed when launching a diffracting beam in an appropriately biased sample, without any appreciable background illumination. As reported in the very first experiments on solitons by Duree et al. in Ref.[7], the beam is observed to form, after a time interval of the order of 10ms, a spatial soliton, remain almost stationary for an interval of 20ms, and then decay into a once again diffracting beam. This peculiar sequence, which involves a rare plateau evolution, is referred to as a quasi-steady-state soliton.
The time dependent version of Eq.(2) truncated at zero order in η reads

$$\frac{\partial Y}{\partial \tau} + QY = G, \tag{11}$$

where $\tau = t/\tau_d$, $\tau_d = \epsilon_0 \epsilon_r \gamma N_a/(q\mu s(N_d - N_a)I_b)$ is the characteristic dielectric time constant, γ is the recombination rate, μ the electron mobility, s the donor impurity photoionization efficiency, and, as occurs for most configurations of interest to soliton dynamics, the charge recombination time τ_r=$1/(N\gamma)$ is much shorter than charge displacement dynamics, and no time dependence in the boundary conditions are considered (G=-1). For small modulation, Eq.(11) gives the characteristic exponential dynamics with $t = \tau_d/Q$, but as soon as Q is spatially resolved, and furthermore is itself evolving, a continuum of different time constant contribute. Seen in a different perspective, soliton time evolution is highly time-nonlocal, as the formally equivalent integral version of Eq.(11)

$$Y = e^{-\int_0^\tau Q d\tau'} \left(1 + \int_0^\tau d\tau' e^{\int_0^{\tau'} Q d\tau''} \right), \tag{12}$$

indicates [54]. The full complexity of this behavior emerges during transients, i.e., when I changes in an appreciable manner with time. This occurs most evidently during the very first collapsing stage, for times $\tau \leq 1/(1+u_0^2)$, and leads to a stretched exponential evolution [63]. A characteristic of multi-scale processes, this behavior is common to the entire family of soliton supporting cumulative nonlinearities. A numerical approach to Eq.(12) coupled to the parabolic wave equation confirms experimental findings, but to date there is no clear understanding as to why the soliton should pass through a plateau, and, more importantly, how to evaluate the so-called threshold nonlinearity for which self-trapping is achieved, the duration of the plateau, and the nonlinear equation, such as Eq.(8), for which the waves are eigenfunctions. Furthermore, we do not have a means to predict the actual trapping Δx at the plateau, for a given nonlinearity.

In order to highlight the sole role of high modulation depth, we can considerably simplify our predictions, excluding nonlocality, by imposing that Q *not* have relevant time-dependence [64]. In this case Eq.(12) is simplified to give

$$Y = e^{-\tau\overline{Q}}(1 + \frac{1}{\overline{Q}}(e^{\tau\overline{Q}} - 1)), \tag{13}$$

where $\overline{Q}$ is an appropriate time average. Whereas this approach can be meaningful and useful for conditions in which a soliton (i.e., the beam I) is steady, such as for steady-state incoherent solitons which we will describe below, it can say nothing as to *beam* transients. It has been speculated that Eq.(13) could be valid when a negligible amount of diffraction is involved [65, 66, 67, 68, 69].

Delaying the discussion of incoherent self-trapping, a consequence of cumulative inertial response, to section 10, we add a mention to the wealth of transient phenomenologies that occur for higher-dimensional needles [70], and those associated to a time-dependent external bias E_0 [11, 71, 72, 73]. Lastly, we should mention the study of *single* pulse propagation and space-charge build-up [74, 75].

6 Various photorefractive mechanisms supporting self-trapping

One of the nicest features of self-trapping of optical beams in photorefractives is the diversity of mechanisms that can support solitons. Apart from the solitons described in the previous section, which relied on an externally-applied bias field, self-trapping in photorefractives can also arise from photovoltaic effects [76], or from diffusion-driven effects [77], or from resonantly-enhanced effects caused by the excitation of both electrons and holes [78]. In several cases, combinations of two of these effects can also lead to solitons [e.g., solitons supported by the photovoltaic and the screening nonlinearities simultaneously]. Furthermore, in some cases, self-trapping can arise from semi-permanent changes in the crystalline structure, either though clustering of ferroelectric domains [79], or through re-poling of macroscopic regions [80, 81], both being driven by the local space charge field. Such permanent changes are in fact "fixed" (soliton-induced) waveguides, acting as microstructure optical circuits "impressed" into the volume of the bulk nonlinear crystal. To this date, this is one of the very few techniques to create intricate 3D optical circuitry. In this section, we briefly review these additional mechanisms supporting self-trapping of optical beams in photorefractive media.

6.1 Photovoltaic solitons

Soliton-supporting mechanisms appear in photorefractives also in the absence of electric fields, the major example being photovoltaic solitons [76, 82]. Here, in open-circuit conditions and for the (1+1)D geometry, the non-uniform optical excitation translates into a non-uniform photoinduced current. This, at steady state, must be countered by the drift of photo-excited charge (electrons) in response to E. Under conditions analogous to those leading to Eq.(4), for a beam with features Δx of the order of several microns, we arrive again at a saturable nonlinearity

$$\Delta n(I) = -\frac{1}{2} n^3 r_{33} E_p \frac{I}{I_d + I} = -\Delta n_{0,p} \frac{I}{I_d + I}, \tag{14}$$

where $E_p = \beta_{ph} N_a \gamma/(q\mu s)$, I_d the equivalent dark illumination, the constitutive relation for the current along x being $J = q\mu N E + \beta_{ph}(N_d - N_d^+)$. Much in the same fashion of Eq.(8), this nonlinearity leads to a nonlinear soliton equation that supports bright and dark solitons on the basis of the sign of $\Delta n_{0,p}$, i.e., on the sign of β_{ph} [76, 82].

Most of the experiments with photovoltaic solitons have been carried out in $LiNbO_3$, for which β_{ph} is negative for an extraordinary beam propagating along z, this leading to the observation of one-dimensional dark photovoltaic solitons [83].

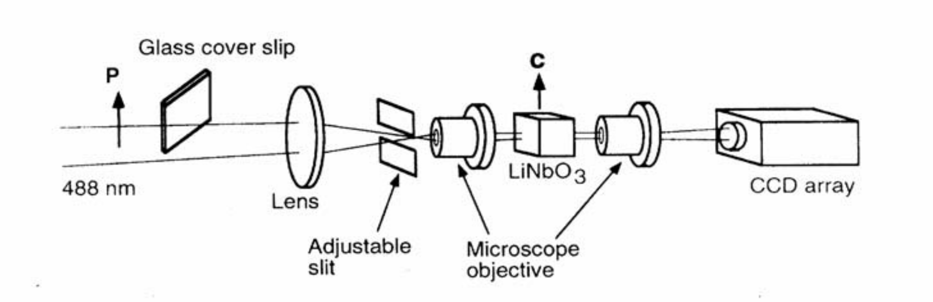

Fig. 14. Photovoltaic dark soliton setup, from [83]. Note the use of a resolved transverse phase-structure obtained by having part of the launch beam pass through an appropriate piece of glass.

As occurs for the screening type nonlinearity, photovoltaic solitons can also form in the higher-dimensional case. For $LiNbO_3$, these form as dark vortex solitons which are supported by a "spiraling" transverse phase modulation [46] (see Fig.(16)). In KNSBN, photovoltaic self-action is self-focusing, and even bright (2+1)D photovoltaic solitons have been detected [38]. Moreover, it has been predicted and demonstrated that the use of a background illumination, not a strict requirement in photovoltaics, allows the transitions from the defocusing to a focusing nonlinearity in $LiNbO_3$ [84].

6.2 Resonantly-enhanced self-trapping in semiconductors

One of the nicest features of photorefractive solitons is the very low power level at which they can form, allowing soliton experiments with microwatt

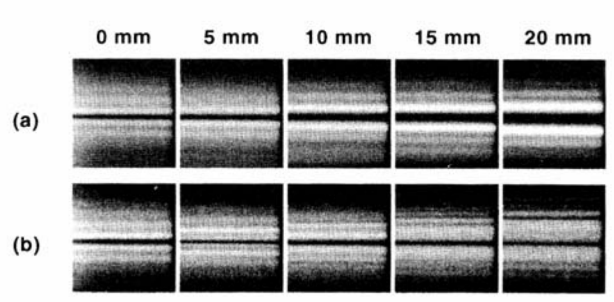

Fig. 15. Observations of dark photovoltaic solitons, from [83]. (a) linearly diffracting intensity distribution of a dark notch; (b) self-trapped intensity distribution, for various propagation distances in the sample.

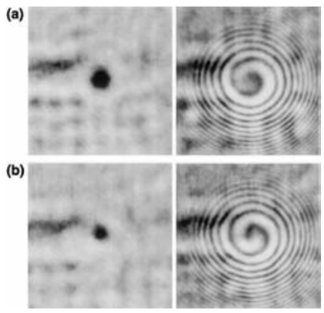

Fig. 16. Self-trapping of a photovoltaic vortex, from [46]. (a) output intensity distribution before self-focusing begins, and phase pattern; (b) output intensity distribution trapped by the photovoltaic field, and phase pattern.

(and lower) power levels. As will be discussed in the section on applications, photorefractive solitons, and the waveguides they induce, combine properties that suggest interesting applications ranging from reconfigurable directional couplers, beam splitters, waveguide switching devices, tunable waveguides for second harmonic generation, and highly efficient optical parametric oscillators in soliton-induced waveguides. In general, however, the formation time of solitons in most photorefractive materials is rather long, except when very high intensities are used [32]. This is because the photorefractive nonlinearity relies on charge separation, for which the response time is the dielectric relaxation time, i.e., inversely proportional to the product of the mobility and the optical intensity, and the mobility in photorefractive oxides is low. In principle, however, photorefractive semiconductors, (e.g., InP, CdZnTe), have a high mobility and could offer formation times a thousand fold faster than in the other photorefractives. However, the electrooptic effects in these semiconductors are tiny, which implies that solitons that are as narrow as 20 optical wavelengths necessitate very large applied fields, making solitons in them almost impossible to observe. But, in some of these materials (InP and CdZnTe) that have both holes and electrons as charge carriers, a unique resonance mechanism can greatly enhance the space charge field by as much as 10 times (and more) over the applied electric field. This enhancement yields large enough self-focusing effects that can support narrow spatial solitons. The resonant enhancement of the space charge field has led to the observation of solitons in photorefractive InP [78, 35] and CdZnTe [85].

As mentioned, the resonant enhancement of the space charge field occurs in materials with both types of charge carriers, both being excited from a common trap level: one excited optically and the other excited by temperature (or by a second optical beam of a longer wavelength). These two excitations work in opposing fashions: one fills (populates) the mid-gap traps whereas the other empties them. At steady state, when a focused beam illuminates a biased crystal of this kind, and the beam intensity is such that the photoexcitation rate of one type of carrier is comparable to the thermal excitation rate of the other type of carrier, the concentration of both free carriers at the illuminated region decrease drastically. The intuitive explanation is as follows [86]. Under proper conditions, the ratio between the concentrations of electrons and holes is equal to their ratio in the absence of light, and thus has a constant (coordinate-independent) value (see appendix of [17]). The net excitation rate of the traps is the difference between the thermal (holes) and optical (electrons) excitation rates. At resonance, the net excitation rate goes to zero. At the same time, at steady state the excitation rate must be equal to the recombination rate, which, in turn, is proportional to the free charge concentration. Hence, at resonance (when the excitation rates of holes and electrons are comparable) the free charge concentration goes to zero. Consequently, the local electric field is highly enhanced because the current at steady state must remain constant throughout the crystal. For a given temperature, this enhancement occurs at a specific intensity (the resonance intensity), for which the thermal and optical excitation rates are comparable. It is a resonant enhancement, although it is an intensity-resonance and not an atomic resonance. The enhanced electric field compensates for the smallness of the electrooptic coefficient and enables a sufficiently large change in the refractive index to support narrow solitons.

The observation of solitons in photorefractive semiconductors [[78, 35, 85]] is especially important for several reasons. First, the solitons are generated at optical telecommunications wavelengths. Second, they facilitate microsecond soliton formation times even at very low (microwatt) optical power. These suggest that optical spatial solitons could form from light beams emerging from ordinary optical fibers (conventional data-transmission lines) on nanoseconds time scales, and could be all-optically switched on-off by employing the intensity resonance in photorefractive semiconductors.

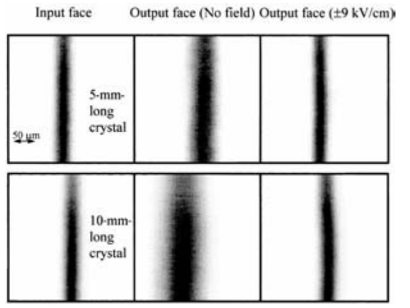

Fig. 17. Observed transverse intensity distribution for (1+1)D self-trapping in InP, from [78].

6.3 Diffusion-driven self-action

In a conventional noncentrosymmetric crystalline phase, such as that characterizing SBN or $BaTiO_3$ at room temperature, charge diffusion leads to an asymmetric index profile, which translates into a transverse phase chirp that produces self-bending. As discussed in section 3.1 in conjunction with Eq.(5), diffusion is typically merely a correction for screening soliton studies.

Consider now a situation where no external field is applied. On the basis of Eq.(1) with $g = 0$ (null current), $E = -\frac{k_b T}{q}\frac{1}{I+I_b}\frac{dI}{dx}$. Higher order corrections due to saturation in this case are even less important, such that for a 10 μm beam, they represent a relative import of the order of $\epsilon_r \cdot 10^{-6}$, where ϵ_r is the relative dielectric constant. For a sample heated above the ferroelectric-paraelectric phase-transition, manifesting a *quadratic* electro-optic response, the resulting nonlinearity leads to a *symmetric* lensing effect, of the type $\Delta n(I) \propto (\frac{1}{I+I_d}\frac{dI}{dx})^2$. Although in most conditions, such self-action is negligible, in the very proximity of the phase-transition, where ϵ_r attains values of the order of 10^4, self-focusing, the precursor of soliton formation, has been observed [77, 87]. The resulting nonlinear equation, which can be extended also to the full (2+1)D case, represents the singular situation in which a nonlocal nonlinearity (involving a spatial derivative) allows for the explicit analytical prediction of both the observed nonlinear diffraction, along with such novel effects as ellipticity recovery (see Fig.(18)), and the prediction of a full family of solitons, which, however, requiring extremely pure samples and precise thermal conditions, and have not been observed.

6.4 Fixing the photorefractive soliton: Self-trapping by altering the crystalline structure

Solitons in photorefractives are typically supported by the linear polarization response to the local space-charge field, $\boldsymbol{P} = \epsilon \boldsymbol{E}$. However, optical beams can also self-trap in photorefractiue media by altering the crystalline structure

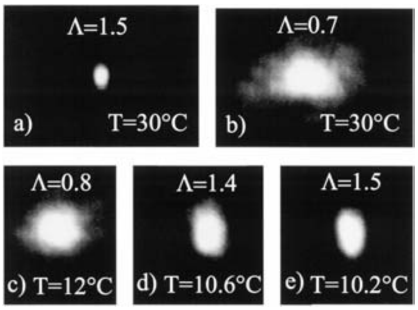

Fig. 18. Diffusion-driven ellipticity recovery. The input ellipticity (a) Λ is recovered at the output (b-e) as the crystal temperature T is brought closer to the Curie temperature, enhancing diffusion-driven effects, from [87].

of the nonlinear medium in which the beam propagates. This happens when the local space charge field E becomes comparable with the coercive field E_c, and is directed in a direction different from the poling direction. For low-modulation gratings, such conditions rarely emerge. But photorefractive solitons, being entities with an inherently high modulation depth, are always associated with locally high electric fields that can readily depolarize a sample. Consider a screening soliton, which exists as a consequence of an external bias E_0 directed along the poling axis x. Once the soliton has formed, charge has redistributed so as to screen, at least in part, the field. This means that across the beam profile, charge distribution engenders a field that is approximately opposite the bias. Removing the illumination (and the background) and switching the external bias off unravels this field. The result is that $E_{V=0} \simeq -E_0$ in the region that before hosted the soliton. In SBN Klotz et al. have shown how this field can, not only depolarize the sample, but actually permanently fix the waveguiding structure that accompanies the soliton, an achievement that can have considerable impact in soliton based devices [80, 81].

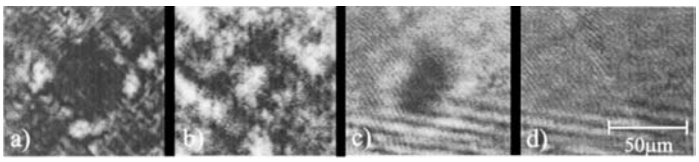

Fig. 19. Domain and cluster structure induced by a spontaneous self-trapped process in metastable KLTN, from [79]. (a-b) Underlying domain structure as seen with the two principal polarizations; (c-d) melting away of domains into clusters, clusters into a homogeneous structure, as temperature is increased from the Curie point.

Ferroelectric clusters can also play a more dynamic role in spatial self-trapping, when they interplay with light-generated space-charge during soliton formation. This has been observed in a metastable paraelectric, in proximity of the transition temperature, where charge-diffusion fields become comparable to E_c [79]. The result is a complicated optical-domain dynamic which also leads, in appropriate conditions, to a form of spatial self-action known as spontaneous self-trapping (see Fig.(19)). The description of these processes, which requires the modelling of domain formation, and their light-matter interaction throughout propagation, is beyond the linear polarization approach. Such a theory has yet to be formulated. Nevertheless, the phenomenology seems to indicate a more general behavior of propagation in metastable hosts.

7 Alternative photorefractive materials

Research into nonlinear beam propagation in photorefractives is carried out, in parallel, in a series of relatively different materials, each presenting a slightly modified version of the mainstream screening type nonlinearity, with its advantages and setbacks. Amongst these, we should mention the sillenites, such as BTO, BSO and BGO, paraelectrics, such as KLTN, and photorefractive polymers [88, 89] and organic gels [90].
Some of the original research in photorefractive self-trapping was carried out using nonferroelectric sillenites, in a configuration similar to that used for ferroelectric samples [16, 91, 92, 93]. The main difference between the physical processes is that sillenites present a fairly strong natural optical activity, which leads to polarization rotation (during propagation) of both the self-focused and the background beams. This causes the effective nonlinearity to change in z. Strictly speaking, solitons (diffraction-free) beams cannot exists in such optically-active materials [94, 95, 96], at least not in the typical scheme of screening solitons. However, using very short samples (a typical value of optical activity is $\rho_0 \simeq 6°/mm$), and employing pre-compensation (in which the beams are not extraordinarily- and ordinarily-polarized, but evolve to these halfway through the sample), does allow the observation of quasi-solitons. Another setback in using sillenite crystals is in the relatively weak effective electro-optic coefficient $r_{eff} \simeq 0.06pm/V$ (for BTO), which implies the use of very high bias fields. Altogether, the sillenite crystals have served an important role in the first explorations of steady-state self-focusing [16], but have been rarely used for solitons experiments since then, simply because other materials offer more favorable conditions for soliton generation.
Part of screening soliton studies are carried out in paraelectrics, where the quadratic electro-optic effect supports a screening nonlinearity in all similar to the noncentrosymmetric counterpart, of the form $\Delta n \propto (E_0)^2/(1{+}I/I_b)^2$ [97].
In these, almost all the conventional photorefractive soliton phenomenology can be observed, from (1+1)D [33] to (2+1)D solitons [36]. Dark solitons have

yet to find an appropriate material, because in such crystals the nature of the index change cannot be reversed upon reversal of the applied field, as is the case for noncentrosymmetric crystals with the linear electrooptic effect. Given the generally weak electro-optic response in the high-symmetry phase, most studies are carried out in proximity of the ferroelectric-paraelectric transition: the crystals must be thermalized to stabilize their dielectric response.

8 Soliton interaction-collisions

Nonlinearity generally allows the coupling and energy exchange among beams and modes. For solitons, nonlinearity not only supports their propagation-invariant nature, but leads to a unique coupling dynamic in which the single solitons behave as quasi-rigid particles when they are made to interact with one another. In fact, this particle-like dynamic is the very reason for the term "soliton". In this spirit, interactions between solitons are commonly referred to as **soliton collisions**, and constitute the most fascinating features of soliton phenomena. Soliton interactions can be generally classified into coherent and incoherent interactions. **Coherent interactions of solitons** occur when the nonlinear medium can respond to interference effects that take place when the beams are overlapped. They occur for all nonlinearities with an instantaneous (or extremely fast) time response (e.g., the optical Kerr effect and the quadratic nonlinearity). For all other nonlinearities that have a fairly long response time (e.g., photorefractive and thermal), the relative phase between the interacting beams must be kept stationary on a time scale much longer than the response time of the medium. When this occurs, the material responds to interference between the overlapping beams and they exert on each other attractive or repulsive forces, depending on the relative phase between the beams. **Incoherent interactions**, on the other hand, occur when the relative phase between the (soliton) beams varies much faster than the response time of the medium. In this case the resultant force between such bright solitons is always attractive. Altogether, soliton interaction-collisions are universal, exhibiting the same basic features in spite of the widely diverse physical origins for the self-trapping. For a detailed review on soliton interaction forces, see [3].

Photorefractive solitons play an especially important role in the study of soliton interactions, and have here greatly contributed to soliton research at large. This leading role is a consequence of a series of factors: the relative ease in soliton generation, which lowers the complexity of multiple soliton supporting schemes; the saturable nature of the nonlinearity, which offers many features that are simply non-existent with ideal Kerr-type solitons; the availability of both (1+1)D and (2+1)D solitons, in a 3D bulk environment; and the relatively slow response of photorefractive materials which facilitates the possibility of studying both coherent and incoherent collisions in the same material system. Over the past eight years, many groups have come to realize the wealth of possibilities offered by soliton interaction studies in photorefractives, and consequently many of the soliton collision features were first demonstrated with photorefractive solitons.

The physical intuition behind soliton collisions relies on the generic idea that a soliton is a bound state of its own induced potential, or, in optics, a guided mode of its own induced waveguide. Having this in mind, one can understand soliton collisions by comparing the collision angle to the (complementary) critical angle for guidance in the single soliton-induced waveguides (that is, to the angle with the propagation axis below which total internal reflection occurs). Whether or not energy is coupled from one soliton into the waveguide induced by the other soliton, depends on the relation between the collision angle and that critical angle. In terms of a "potential well", capture depends on whether the kinetic energy of the colliding wave-packets results in a velocity that is smaller than the escape velocity. If the collision occurs at an angle larger than the critical angle, the solitons simply go through each other unaffected - very similar to the behavior of Kerr solitons (the beams refract twice while going through each other's induced waveguide but cannot couple light into it). If the collision occurs at "shallow" angles, the beams can couple light into each other's induced waveguide. Now if the waveguide can guide only a single-mode (a single bound state), the collision outcome will be elastic, essentially very similar to that of a similar collision in Kerr media (with the exception that now some very small fraction of the energy is lost to free radiation). However, if the waveguide can guide more than one mode, and if the collision is attractive, higher modes are excited in each waveguide and, in some cases, the waveguides merge and the solitons fuse to form one single soliton beam. Such a fusion process is always followed by a small energy loss to radiation waves, much like inelastic collisions between real particles. This naive picture of soliton interactions gives qualitative understanding of the complex behavior of soliton collisions, yet it is incomplete. In reality, the interacting solitons affect each other's induced waveguide, and the true collision process is much more complicated.

The first experimental papers on collisions between photorefractive solitons were also the first papers reporting fusion of solitons in any medium [49, 98] (in parallel to a similar observation with solitons in atomic vapor, for which the nonlinearity is also saturable). These experiments reported on incoherent collisions, between (2+1)D [49] and between (1+1)D solitons [98], during which at large collision angles the solitons passed through one another unaffected, whereas at shallow collision angles they fused to one another. Following these experiments, a team headed by W. Krolikowski studied coherent collisions and demonstrated fission of solitons ("birth" and annihilation of solitons [50, 51], which also constitutes the first observation of such effects in any solitonic system. Other groups have followed and mapped out coherent interactions between (1+1)D and (2+1)D solitons [99, 100]. All of these were collisions between solitons launched with trajectories in the same plane. However, because the photorefractive nonlinearity is saturable, one can also look at collisions of (2+1)D solitons with trajectories that also do *not* lie in the same plane: full 3D interactions. When non-parallel solitons with trajectories that do not lie in the same plane are launched simultaneously, they interact (attract or repel each other) via the nonlinearity and their trajectories bend. The system possesses initial angular momentum: if the soliton attraction exactly balances the "centrifugal force" due to rotation, the solitons can "capture" each other into orbit and spiral about each other, much like two celestial objects or two moving charged particles. This idea was suggested in the context of coherent collisions. However, because coherent interactions are critically sensitive to phase, in this case solitons can never attain stable orbits, and spiral about each other. Instead, they always either fuse to form a single beam, or "escape away" from each other. On the other hand, the purely attractive nature of the force between incoherent solitons and its independence of the relative phase of the two interacting solitons makes these ideal for the orbiting observation. Under proper initial conditions of separation and beam trajectories, solitons indeed capture each other into an elliptic orbit [48]. If the initial distance between the solitons is increased, their trajectories slightly bend toward each other, but their "velocity" is larger than the escape velocity, and they do not form a "bound pair". On the other hand, if their separation is too small, they spiral on a "converging orbit" and eventually fuse. In reality, the 3D spiraling-interaction mechanism is much richer and more complicated than initially thought. It turns out that the two spiraling-interacting solitons exchange energy by coupling light into each other's induced waveguide. This is because the nonlinearity is saturable and the trajectories are at shallow angles. But, because the two interacting solitons have equal power, the energy exchange is symmetric. The energy exchanged is, of course, phase-coherent with its "source" but phase incoherent with the soliton into which it was coupled. Thus, even though the solitons are initially incoherent with each other, the energy exchange induces partial coherence and thus contributes to the forces involved. The end result is that the two solitons orbit periodically about each other and at the same time exchange energy periodically, with the two periods (the spiraling period and the energy exchange period) being only indirectly related [101]. This complicated motion is stable over a wide range of parameters [101]. To some extent, whether or not the spiraling can go on indefinitely is yet an open question, because it is possible that, after long enough propagation distances (far beyond the present experimental reach), the solitons eventually merge [102, 103]. Another interesting feature of spiraling-interacting solitons is the fact that if one adds a tiny seed in one of the input solitons that is coherent with the other soliton, the relative phase between these coherent components (the seed and the other soliton) controls the outcome of the collision process, and can turn a spiraling motion into fusion or repulsion [101]. Altogether, the observation of spiraling-interacting solitons has introduced new concepts to soliton physics: not only energy (power) and momentum are conserved, but also the conservation of angular momentum, which is the fundamental symmetry that enables spiraling. These pioneering experiments are a characteristic example of the contribution of photorefractive solitons to nonlinear science.

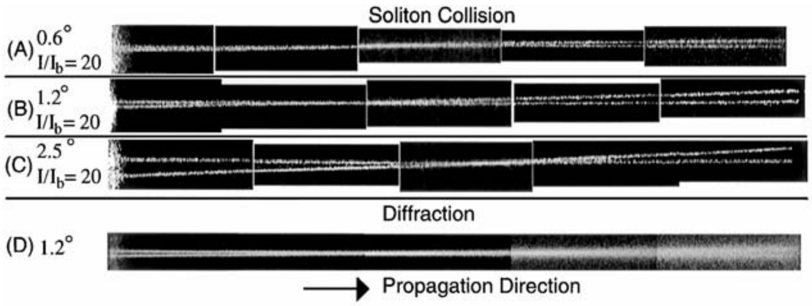

Fig. 20. Collisions, from [49]. Top view of colliding incoherent diffracting beams and solitons for different relative angles.

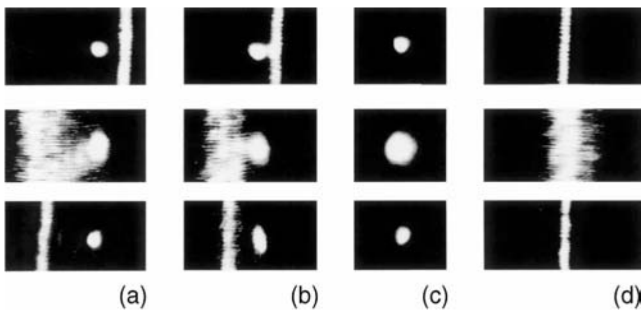

Fig. 21. Hybrid-soliton collisions, from [104]. (a) input launch of a two-dimensional and a one-dimensional diffracting beam, output intensity distribution with no applied field, output intensity distribution with applied self-trapping field; (b) same as (a) but for a smaller collision angle; (c-d) single beam self-trapping in the same experimental conditions.

It is interesting to compare soliton and plane-wave interaction phenomenology in a photorefractive. The crossing of two plane waves, even at low angles of the order of fractions of a degree, leads to energy transfer, which, even if mediated by space-charge components of the order of η, brings to finite if not total coupling from one beam to the other. Two solitons, on the other hand, behave in a diametrically opposite manner: they cross each other without appreciable energy transfer, even when they are mutually coherent.

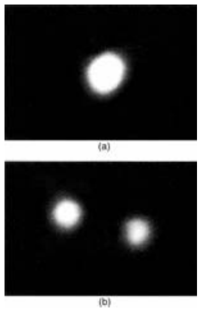

Fig. 22. Soliton fusion, from [50]. Two identical solitons fuse at the output of the sample when they are in-phase (a), whereas they repel when they are out-of-phase (b).

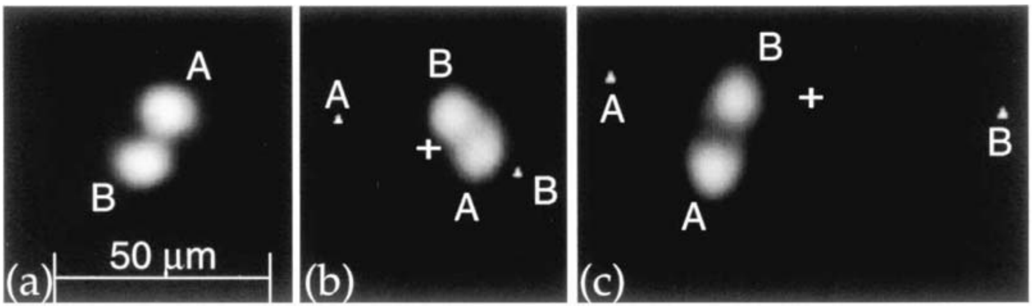

Fig. 23. Soliton spiralling, from [48].(a) Beams A and B at the input face of the crystal, (b) the spiraling soliton pair after 6.5 mm of propagation, and (c) after 13 mm of propagation. The centers of diffracting A and B are marked by white triangles. The white cross indicates the center of mass of the diffracting beams A and B in (b) and (c).

The conundrum has a straightforward geometrical explanation: even though no soliton mechanism forbids the formation of a resonant coupling grating in the region where the two beams overlap, this overlap is spatially limited by the mutual angle θ in their propagation direction and their very narrow width Δx, which is typically no more than 20 wavelengths wide. Thus, in two-wave-mixing terminology, even though the coupling coefficient γ can be reasonably high (e.g., at angles corresponding to the Debye length), the equivalent energy transfer γL, where L is the effective interaction distance in the sample, is negligible. Altogether, since self-lensing is a purely phase-modulating mechanism without energy coupling to new modes, soliton interactions, even in photorefractives (which could give rise to energy transfer driven by the diffusion field) is well described by means of soliton interaction forces based on wave-overlap integrals, and the intrinsic energy coupling mechanism that acts in two-wave-mixing is absent.

In the spirit of driving nonlinear soliton studies to their most extreme consequences, photorefraction forwards a unique possibility of studying hybrido-dimensional soliton collisions [104]. Namely, testing to what extent a soliton undergoes particle-like dynamics even during a collision between two solitons of different dimensionality. The possibility of carrying out these studies arises from the saturable nature of the nonlinearity, which facilitates stable (2+1)D solitons, as well as the quasi-stable propagation of (1+1)D solitons in 3D bulk media. Strictly speaking, the latter case is not stable in the absolute sense, and would, after a sufficiently large propagation distance, suffer from transverse instabilities and disintegrate into an array of 2D filaments. However, if the (1+1)D soliton is launched with parameters close enough to the soliton existence curve, and if its parameters are chosen such that the operation point is in the saturated regime, the "instability length" is very long, and for all practical purposes such a (1+1)D soliton propagates in a stable fashion in the bulk. These conditions have indeed allowed to experimentally observe the collision between a "needle soliton" and a "stripe soliton" [104], resulting in a new avenue in the investigation of particle-like behavior of solitons at large.

A more recent series of studies targeted collisions of solitons propagating in opposite directions [105, 106, 107, 108]. The concept actually applies to any soliton-supporting system and introduces a profoundly different scenario for interacting solitons [109]. Experimentally, thus far only incoherent collisions between such solitons have been studied [106, 108], whereas the more interesting case of coherent collisions [109, 107] is still unexplored [apart for the very specific case of a vector soliton formed by counter-propagating fields which has been recently demonstrated [110]; but this is not really a vector soliton, and not a soliton collision experiment]. In photorefractives, however, during an almost head-to-head collision, the interaction between the solitons occurs along a spatially extended region, hence spatially nonlocal effects, such as self-bending driven by diffusion fields, have an important impact on the collision dynamics. The experiments of [106, 108] show that the soliton collision dramatically modifies the self-bending of both solitons, in a way that can be directly utilized for all-optical beam deflection, steering and control.

9 Vector and composite solitons

In their most basic form, solitons are a coherent entity described by a single optical wavefunction. The simplest (so-called scalar) soliton occurs when the soliton constitutes of a single field, which populates the lowest mode of its own induced-waveguide. A more complex soliton, a vector soliton, occurs when more than one field populates the lowest mode. These were first suggested by Manakov in Kerr media, when self- and cross-phase modulation are equal. Vector solitons, however, can be also composite: they can be composed of optical fields that populate *different* modes of their jointly induced-waveguide. In one realization, composite solitons are made of a bell-shaped (bright) component populating the lowest mode and a dark component being the second mode. A more interesting situation occurs when the field constituents of the composite-soliton populate different bound-modes of their jointly-induced potential. These composite, multi-mode, solitons can have two or more intensity humps, and can appear in (1+1)D and in (2+1)D, in a variety of intriguing combinations, including (2+1)D composite solitons carrying angular momentum within their field constituents. As will be explained below, almost all the experiments with vector solitons, and practically all the experiments with multi-mode solitons, were carried out in photorefractives.

A key issue regarding a vector soliton is that interference terms between the soliton constituents should not contribute to the nonlinear index change (otherwise the induced potential would vary periodically during propagation). Thus, the field constituents making up a vector soliton could originate from orthogonal polarizations states, or from fields at widely-spaced frequencies. In the polarization-based technique, there are no interference terms, whereas in the widely spaced frequencies method, the interference terms are not synchronized with either of the soliton constituents. Photorefractives, however, have offered a much more appealing technique to generate vector solitons: making up a vector soliton from field constituents that are incoherent with one another, that is, their relative phases are randomly fluctuating. When the relative phase between the fields making up the soliton fluctuates much faster than the response time of the nonlinearity, their contribution to interference terms averages out to zero. This method, suggested by Christodoulides [111], has revolutionized the area of vector solitons. With this mutual incoherence method, first a degenerate (Manakov-like) soliton was observed [112], the same year that the first Manakov-type solitons were observed in Kerr media. This was followed by an observation of a vector soliton made up of a bright and a dark component [113]. Shortly thereafter, the first multi-mode/multi-hump solitons were observed [114]. In this multi-mode soliton experiment, the two input field distributions resembled the first and/or second and third modes of a slab waveguide. Interestingly, the experiment has also shown that it is possible to observe multi-mode solitons made up of solely higher modes (the second plus the third modes, trapped in their jointly-induced potential [114]). That is, the experiment has indicated that multi-mode solitons can exist and propagate in a stable fashion for distances much larger than the diffraction length, in spite of the fact they are made up of excited states only. Several years later, (2+1)D dipole-type composite solitons were also demonstrated experimentally [39, 41]. These vector solitons consist of a bell-shaped component jointly trapped with a 2D dipole mode. The ability to generate (2+1)D composite solitons opens up a whole new range of possibilities that has no counterpart in a lower dimensionality. One fascinating example is the recently observed rotating "propeller" soliton [40]. This is a composite soliton made of a rotating dipole component jointly trapped with a bell-shaped component. It carries and conserves angular momentum, although its constituents exchange angular momentum as they propagate. The soliton turns out to be generically robust, so much that even during inelastic collisions with other composite solitons, the collision products are predicted to be also composite rotating propeller solitons [115].

The relatively ease with which vector and composite solitons are generated in photorefractives, by employing the mutual incoherence technique, has also led to a series of experimental efforts demonstrating interaction-collisions between vector solitons. It turns out that temporal optical vector solitons, and non-optical vector solitons are very difficult to generate, hence the collision experiments with optical spatial vector soliton were truly pioneering. For example, it has been predicted, more than two decades ago, that collisions between Manakov-like vector solitons give rise to a symmetric exchange of energy between the soliton constituents. This elegant phenomenon was observed only recently, with photorefractive vector (Manakov-like) solitons [116, 117]. The energy-exchanges between the soliton components (which have nothing to do with photorefractive two-wave-mixing) have direct implications in a new form of reversible computing, in which a "state" is coded as the ratio between the soliton components [118]. In this scheme, computation is performed through the energy-exchange interactions (in which the "states" change) during collisions between vector solitons. The experiments have shown that indeed, not only such symmetric energy exchanges do occur, but also information can be transferred through a series of cascaded collisions between vector solitons [116, 117].

Finally, photorefractives were also the means for experimental studies of interaction-collisions between multi-mode solitons, in which shape transformations were observed [119]. These were the first ever experiments with collisions of multi-mode solitons.

The general ideas behind multi-component vector solitons proved invaluable for later developments and in particular to the area of incoherent solitons discussed in the next section.

10 Incoherent solitons: self-trapping of weakly-correlated wavepackets

Until 1996, the commonly held belief was that all soliton structures should be inherently coherent entities. In that year however, an experiment carried out at Princeton demonstrated beyond doubt that self-trapping of a partially spatially-incoherent light beam [42] is in fact possible, if the nonlinearity has a non-instantaneous temporal response. In that experiment, the optical beam was quasi-monochromatic, but partially spatially-incoherent and the nonlinear medium was photorefractive, with a response much slower than the characteristic time of the phase fluctuations in the incoherent beam. The resultant self-trapped beam is now commonly referred to as an "incoherent

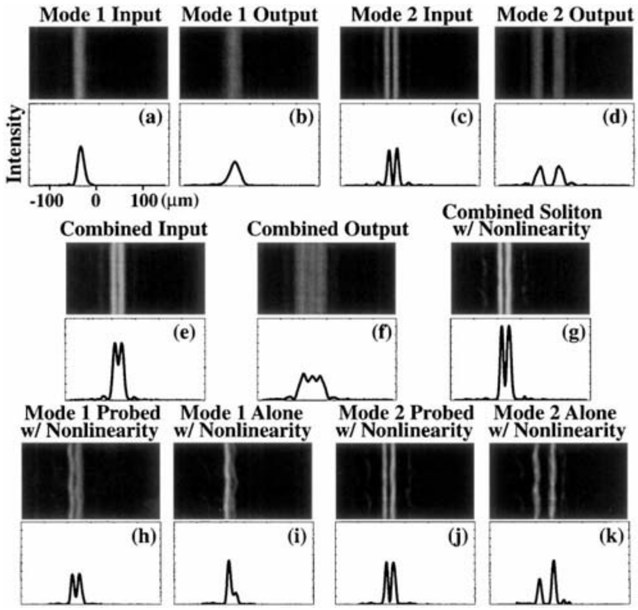

Fig. 24. The observation of a multimode soliton, from [114],with mode 2/mode 1 = 1.0 (a),(b) input (18 μm FWHM) and linear diffraction (27 μm) of mode 1; (c),(d) input (28 μm) and linear diffraction (62 μm) of mode 2; (e),(f) combined input (26 μm) and combined linear diffraction (58 μm); (g) composite soliton formed (26 μm) with application of 800 V/4 mm; (h) first mode obtained after quickly blocking second mode; (i) steady state of first mode alone with nonlinearity on; (j) second mode obtained after quickly blocking first mode; and (k) steady state of second mode alone with nonlinearity on.

soliton" or a "random-phase soliton". One year later, the same group observed a similar self-trapping effect with white light from an incandescent light bulb emitting light with a wide 380-720 nm spectrum of wavelengths. This was the first observation of a self-trapped beam made of light both temporally and spatially incoherent: a white light soliton [43]. These two experiments have set the basis for a new field of study in nonlinear optics: incoherent solitons. In yet another experiment, self-trapping of dark incoherent "beams", i.e., 1D or 2D "voids" nested in a spatially incoherent beam, was also demonstrated [47].

For self-trapping of an incoherent beam (an incoherent soliton) to occur, several conditions must be satisfied. First, the nonlinearity must be non-instantaneous with a response time that is much longer than the phase fluctuation time across the optical beam. Such a nonlinearity responds to the time-averaged envelope and not to the instantaneous "speckles" that constitute the incoherent field. Second, the multi-mode (speckled) beam should be able to induce, via the nonlinearity, a multi-mode waveguide. Otherwise, if the induced waveguide is able to support only a single guided mode, the incoherent beam will simply undergo spatial filtering, thus radiating all of its power but the small fraction that coincides with that guided mode. Third, as with all solitons, self-trapping requires self-consistency: the multi-mode beam must be able to guide itself in its own induced waveguide (pages 86-125 in Ref.[4]).

The understanding of how such an incoherent soliton can form also raises some intriguing aspects in comparison to other nonlinear phenomena. Conventional nonlinear optical effects are a consequence of strong material response to (intense) optical excitation, but their macroscopic manifestation is generally the result of distributed enhancement effects, in which weak scattering is amplified by the cooperative excitation of large portions of material. This is the basis, for example, for efficient harmonic generation and wave-mixing. The distributed effect is induced by an extended spatial and temporal coherence, itself transferred by the coherence of the coupling waves. The discovery of incoherent solitons has made evident the basic fact that solitons naturally break this scheme. Even though they do result from nonlinearity, their nature has nothing to do with the constructive interference of distributed effects. On the contrary, it is an intrinsically *local* effect where no coherence transfer mechanism intervenes. Thus, for the more diverse non-optical media that support solitons, the very concept of wave is an abstract average envelope of intrinsically uncorrelated underlying motion, that attains physical meaning when the wave-medium interaction does not react to the erratic fluctuations of the wave constituent.

The experiments demonstrating incoherent solitons have taken the solitons community by surprise, because typically, in most of soliton research (also beyond Optics), all experiments were preceded by a theory predicting the main effects. The experiments demonstrated beyond doubt that incoherent solitons indeed exist. Yet, at the time, something quite important was still missing: a theory! Unlike the case of coherent solitons, where the evolution equation can be straightforwardly derived by adding the nonlinearity to the paraxial equation of diffraction, the description of incoherent solitons was far from being clear. The experiments were of course based on insight and intuition, but then again they gave only few clues, if any, as to how one could develop a theory. Only one thing was certain - the theory of incoherent solitons had to be derived from first principles. Within a year, two different theories were developed to describe incoherent solitons: the coherent density theory [120] and the modal theory [121]. The coherent density theory is, by its very nature, a dynamic approach that is better suited to study the evolution dynamics of incoherent solitons, their interactions, instabilities etc, as they occur in experimental set-ups. In this formalism, the incoherent field is described by means of an auxiliary non-observable function from where one can deduce the optical intensity as well as the associated correlation statistics. The modal theory, on the other hand, by virtue of its inherent simplicity became the method of choice in terms of identifying incoherent solitons, their range of existence and correlation properties. One year later, yet another theory was proposed: the theory describing the propagation of the mutual coherence [122]. Interestingly enough, even though at first sight these three theoretical approaches seem to be dissimilar, they are in fact formally equivalent to one another [123]. All describe quasi-monochromatic yet partially spatially incoherent solitons: they explain the behavior of such entities, provide their statistical properties, and predict their behavior as they interact with one another. As such, they became a very powerful analytical and numerical tool. More recently, these theories were expanded to describe white light solitons: solitons that are made of temporally and spatially incoherent light [124, 125].

The pioneering experiments of Refs.[42, 43, 47] that were the first to show the unexpected fact that random-phase solitons can exist with both spatial and temporal incoherence, opened the way for several other important results. These include for example, anti-dark incoherent states [126], elliptic incoherent solitons [127, 128], coherence control using interactions of incoherent solitons [129, 130], and more. Especially noteworthy are the fundamental studies on modulation instability of incoherent waves and spontaneous pattern formation in weakly correlated system. It has been found, theoretically and experimentally, that such weakly correlated systems indeed exhibit modulation instability, and a uniform (homogeneous) partially incoherent wavefront breaks up into an array of soliton-like filaments. However, for this to occur the nonlinearity needs to exceed a threshold value determined by the coherence properties of the waves [131, 132, 133, 134, 135, 136]. This fact stands in sharp contradistinction with coherent systems, for which there is no such threshold for modulation instability (in a coherent self-focusing system, a uniform wave always breaks up into an array of soliton-like structures). Once this threshold is exceeded, the partially incoherent uniform wave breaks up into an array of localized structures, each behaving as an incoherent soliton. These solitonic "breakup products" interact with one another in an incoherent fashion, exerting long-range attraction on each other. After sufficiently large propagation distance, they aggregate and form clusters of fine-scale structures, leaving large voids between adjacent soliton clusters (aggregates of solitons) [137, 138]. More recent work along these veins is the prediction of modulation instability of white light [139], along with its very recent experimental observation [140], and the work on incoherent pattern formation in cavities [141, 142, 143, 144].

Studies on incoherent solitons and incoherent pattern formation have established that these are not some kind of esoteric creatures, specific to photorefractives, but are in fact a general and rich new class of solitons, whose existence is relevant to many other diverse fields beyond nonlinear optics. For example, incoherent modulation instability effects, soliton clustering and incoherent pattern formation, relate to many systems in nature: from clustering in a cooled atomic gas to self-supported "stripes" of electrons in semiconductors, as well as to gravitational-like effects. In fact, the underlying physics relates to any weakly-correlated wave-system having a non-instantaneous nonlinearity. Altogether, it is fair to say that incoherent solitons are most probably the single most important discovery made with self-trapping effect in photorefractive systems. It has introduced a new concept into soliton research, and has many implications beyond optics, into other arenas where random phase waves and nonlinearities coexist.

11 Applications

11.1 Passive devices

The most basic functionality afforded by a soliton in any physical system, is the transfer of a localized energy/information bearing wave perturbation along an otherwise dispersive propagation. For an optical spatial soliton, this translates into the fact that far-field effects are absent, and the spatial resolution, its phase curvature and coherence properties, and the transverse distribution of energy, remain unaltered: a guided propagation in an otherwise bulk and homogeneous medium, the guide itself being induced by the modes it supports.

For photorefractive solitons, the composite nonlinearity that intervenes allows for a series of useful attributes. For one, a photorefractive soliton can passively guide a second beam of longer wavelength, on consequence of the fact that whereas self-trapping is a product of nonlinearity, the index-modulating system is only weakly wavelength dependent, and will act on an infrared beam in much the same manner that it acts on the soliton [145, 146, 147, 148, 149, 150, 151]. Not being of nonlinear origin, this phenomenon does not depend on the intensity of the guided signal, and, in the measure in which the deposited charge displacement which accompanies the soliton does not redistributed, does not even require the presence of the visible nonlinear wave. This property can be used to integrate a material into a fiber or waveguide device without the development of a crystal-specific technique to grow or tailor a waveguide, the sample itself serving either as an electro-optic phase-transducer, the waveguide not being greatly modified by the application of an arbitrary bias for linear electro-optic response, or simply as redirecting component.

Passive waveguiding gives us the opportunity of differentiating between a nonlinear and a linear behavior. Consider, for example, a dark soliton, which consists of a nondiffracting intensity notch engendered by a π phase jump. In the very same conditions that lead to its formation, it can passively guide a bright longer wavelength mode, even though such a photoactive mode (at the shorter wavelength) could *never* self-trap in the same conditions.

Evidently, more complicated passive devices can be demonstrated, such as reconfigurable directional couplers based on two bright solitons formed in close proximity, Y-junctions, along with more complex multisoliton structures and hybrid soliton-fabricated-waveguides systems [152, 153, 154, 155, 156, 157, 158]. Moreover, in conditions in which charge redistribution cannot be avoided, a considerable advantage can be obtained by implementing the ferroelectric fixing of the routing device [159].

11.2 Active devices

A second functionality is based on the fact that photorefractives offer all-optical functionality even at low intensities, even though this is burdened by slow time response [160, 161, 162, 163, 164, 165]. For example, logic operations can be carried out simply by having two solitons interact, or by modulating two different components of a single vector soliton. We might note that a truly all-optical soliton device must imply a partial absorbtion, and this does not generally spouse implementation. A slightly less ambitious alternative is to use the all-optical operation to steer non photoactive signals.

A second form of active device is that for which soliton dynamics are controlled externally by means of a modulation in the supporting physical parameters. Thus for example, the output direction of propagation can be changed by changing the bias voltage, as a consequence of self-bending [21]. Once again, whereas the signal can be delivered in a fast capacity-limited regime, the optical response will be dominated by the photorefractive time constants.

11.3 Electro-optic manipulation

Passive electro-optic effects have been investigated for soliton-deposited space-charge in paraelectric samples [166]. This allows the fast capacity-limited manipulation of optical circuitry through the sole electro-optic effect, much in the same manner as electro-holographic technology.

In order to grasp the phenomenon, note that in a linear electro-optic response, once a soliton has been formed through a self-trapping Δn_s, the application of an arbitrary control external bias in combination with E_0 leads to a $\Delta n(E_{con}) \propto (E + E_{con})$, which simply changes the constant pedestal on which the soliton guiding pattern induced by E is embedded. This means, for example, that passive guiding can be achieved also for a zero applied external field. For a paraelectric, which is characterized by a quadratic response, $\Delta n(E_{con}) \propto (E + E_{con})^2 = E^2 + 2EE_{con} + E_{con}^2$. The second mixed term allows for an electro-optic *distortion* of the pattern, which, not implying charge redistribution, is a purely spatially resolved electro-response. This has allowed the demonstration of a series of beam manipulation devices, culminating in a two-needle switching device [167] (see Fig.25).

11.4 Nonlinear frequency conversion in waveguides induced by photorefractive solitons

The most promising application of waveguides induced by photorefractive solitons is nonlinear frequency conversion. The conversion efficiency in χ^2 processes is proportional to the intensity of the pump beam, so it is desirable to work with very narrow beams. One easy way to achieve that is to use a focused pump beam. However, in a bulk crystal, the more focused a beam is, the faster it diffracts, and diffraction limits the frequency conversion efficiency because as the interacting beams diffract, (1) their intensities decrease, and (2) the phase-matching condition cannot be satisfied across their entire cross section. Therefore, using waveguides for frequency conversion can greatly improve the conversion efficiency. But thus far it has been difficult to fabricate waveguides from most materials that allow for phase matching, and two-dimensional waveguides are especially difficult to make. Now, (2+1)D photorefractive solitons induce 2D waveguides, and almost all photorefractives are highly efficient in χ^2 frequency conversion. In waveguides, phase-matching should take place among the propagation constants of the guided modes, and is typically obtained through birefringence or periodic poling. In a fabricated waveguide, however, the structure is fixed, so tuning techniques rely on varying the temperature, or on lateral translation in structures with several periods of poling parallel to one other. But waveguides induced by photorefractive solitons offer much flexibility because their waveguide structure and propagation axis (with respect to the crystalline axes) can be modified at will and in real time. Working with photorefractive solitons, one can achieve wavelength tunability while avoiding diffraction by simply rotating the crystal and launching a soliton in the new direction. One can also fine-tune the frequency conversion process by changing the propagation constants of the guided modes through varying the intensity ratio and external voltage, allowing tuning with no mechanical movements.

The first step in the direction of nonlinear frequency conversion in waveguides induced by photorefractive solitons was the demonstration of efficient second-harmonic generation [168, 169, 170]. The experiment have shown that the conversion efficiency can be considerably increased [168], and high tunability can be obtained by rotating the crystal [169]. However, a much more important scenario occurs in a soliton-based optical parametric oscillator (OPO). In an OPO, the threshold pump power is dependent on the signal gain per pass through the crystal. To lower the threshold, one has to increase the signal gain per pass. A waveguide that confines the pump beam as well as the signal and idler in a small area is one very effective way to achieve this. Consider a Gaussian beam at the pump frequency launched into a nonlinear crystal and assume that phase matching is satisfied at the waist, located at the input surface. The threshold pump power is proportional to $[z_0 \arctan^2(L/z_0) + \ln^2(1 + L^2/z_0^2)/4]^{-1}$, where z_0 is the Rayleigh (diffraction) length of the beam and L is the crystal length. For a given L, there exists an optimum beam size for minimum threshold pump power, when $z_0 = L/2.84$. However, if a waveguide is used to keep all beams at the same widths throughout propagation in the crystal, the threshold is simply proportional to z_0/L^2, which continues to decrease as we focus the beam more. The minimum threshold is determined by the smallest size of the waveguide that can be made. To estimate the improvement, consider a focused Gaussian beam with a minimum beam waist of 21 mm and a 15 mm long crystal. An OPO constructed in a waveguide has a threshold pump power 60 percent lower than that of an OPO with the same beam waist in bulk. Therefore, in a waveguide OPO the signal gain per pass can be considerably improved, and the threshold pump power can be substantially lowered for the same cavity loss. This generic idea was recently demonstrated with a doubly-resonant optical parametric oscillator in a waveguide induced by a photorefractive soliton [171]. The OPO threshold was considerably lowered by constructing it within a soliton-induced waveguide. This technique should work even better with singly resonant OPOs and it can substantially reduce the threshold pump power when very narrow solitons are employed. For example, using a soliton beam with a beam waist of 8 mm and a 15 mm long crystal, the OPO threshold pump could be reduced to only 3.5 percent of that for an OPO in the same nonlinear medium, using the same mirrors but without the soliton.

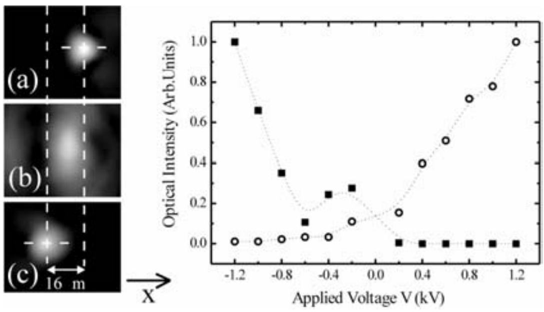

Fig. 25. Electro-optic switching, from [167]

12 New frontiers in photorefractive solitons and concluding remarks

As we hope transpires from the brief review, there is still much to be understood as to the mechanisms underlying spatial photorefractive self-trapping, from the formation of 2+1D solitons to quasi-steady-state solitons, from dark incoherent solitons to spontaneous self-trapping. The field, however, is in constant expansion, and we cannot refrain from mentioning the important

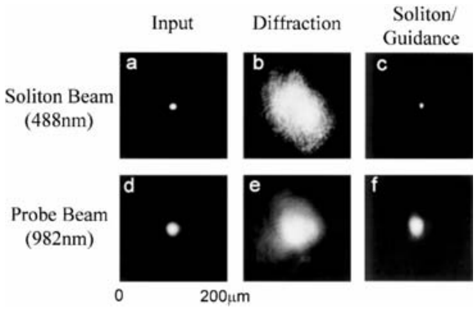

Fig. 26. Second harmonic generation in photorefractive solitons, from [168].

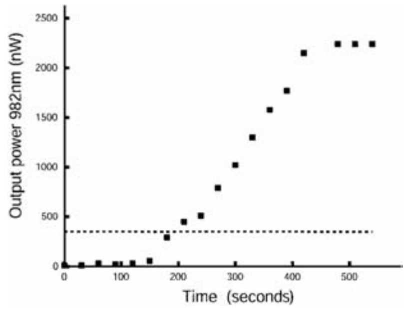

Fig. 27. Second harmonic conversion efficiency increase mediated by spatial solitons, from [168]

achievements in the field of soliton lattices and discrete soliton observation obtained in photorefractive SBN [172, 173, 174, 175, 176, 177], and in the trapping of more exotic excitations such as rings [178].

Although we have attempted to give a detailed account of all aspects of photorefractive self-trapping, the field has undergone such a rapid and extensive evolution that we can hardly guarantee that all contributions have been cited and described. Perhaps this is yet another accomplishment of photorefraction, that is, having given to the soliton science community a powerful and accessible tool with which to further its research and develop new and possibly useful scientific and practical tools.

13 Dedication

Moti Segev would like to dedicate this Chapter to his friend Galit Staier, who was critically wounded in the murderous Palestinian suicide attack on October 4, 2003, in the Maxim restaurant in Haifa, Israel. In that suicide bomb a Palestinian woman blew herself up killing 21 civilians. Galit has has lost her 11-year old son Assaf Staier, her father Ze'ev Almog (71), her mother Ruth

Almog (70), her brother Moshe (43), and his son Tomer Almog (9). Moshe's other son, Oran Almog (11), was critically wounded, while Moshe's wife, Orly Almog, and their daughter Adi Almog (4), were moderately wounded as well. Galit and the survivors of the Staier-Almog family are now slowly recovering. We all hope that Galit will fully recover soon, for her husband (Moti's friend) Ofer Staier, for their son Omri, and for her family and friends.

The Authors join in expressing the hope that the sacrifice of the Staier-Almog family lighten the way to a real peace, free of horrible actions against humanity, such as suicide bombings.

References

1. Segev M., Stegeman G. Self-trapping of optical beams: Spatial solitons Phys.Today 51, 42-48 (1998)
2. Segev M., Optical spatial solitons Opt.Quant.Electron. 30, 503-533 (1998)
3. Stegeman G.I., Segev M., Optical spatial solitons and their interactions: Universality and diversity Science 286, 1518-1523 (1999)
4. Trillo S. and Torruellas W. (Eds.), Spatial Solitons (Springer-Verlag, Berlin Heidelberg 2001)
5. Segev M., Crosignani B., Yariv A., et al. Spatial Solitons in Photorefractive Media Phys.Rev.Lett 68, 923-926 (1992)
6. Crosignani B., Segev M., Engin D., et al. Self-Trapping of Optical Beams in Photorefractive Media J.Opt.Soc.Am B 10, 446-453 (1993)
7. Duree G.C., Shultz J.L., Salamo G.J., et al. Observation Of Self-Trapping of an Optical Beam due to the Photorefractive Effect Phys.Rev.Lett 71, 533-536 (1993)
8. Segev M., Crosignani B., Diporto P., et al. Stability Of Photorefractive Spatial Solitons Opt.Lett. 19, 1296-1298 (1994)
9. Duree G., Salamo G., Segev M., et al. Dimensionality And Size Of Photorefractive Spatial Solitons Opt.Lett. 19, 1195-1197 (1994)
10. Christodoulides D.N., Carvalho M.I., Compression, Self-Bending, And Collapse Of Gaussian Beams In Photorefractive Crystals Opt.Lett. 19, 1714-1716 (1994)
11. Tosi-Beleffi G.M., Presi M., DelRe E., et al. Stable oscillating nonlinear beams in square-wave-biased photorefractives Opt.Lett. 25, 1538-1540 (2000)
12. Segev M., Valley G.C., Crosignani B., et al. Steady-State Spatial Screening Solitons In Photorefractive Materials With External Applied-Field Phys.Rev.Lett. 73, 3211-3214 (1994)
13. Singh S.R., Christodoulides D.N., Evolution Of Spatial Optical Solitons In Biased Photorefractive Media Under Steady-State Conditions Opt.Commun. 118, 569-576 (1995)
14. Christodoulides D.N., Carvalho M.I., Bright, Dark, And Gray Spatial Soliton States In Photorefractive Media J.Opt.Soc.Am. B 12, 1628-1633 (1995)
15. Segev M., Shih M.F., Valley G.C., Photorefractive screening solitons of high and low intensity J.Opt.Soc.Am. B 13, 706-718 (1996)
16. Castillo M.D.I., Aguilar P.A.M., Sanchezmondragon J.J., et al. Spatial Solitons In Photorefractive Bi12Tio20 With Drift Mechanism Of Nonlinearity Appl.Phys.Lett. 64, 408-410 (1994)

17. Shih M.F., Segev M., Valley G.C., et al. Observation Of 2-Dimensional Steady-State Photorefractive Screening Solitons Electron.Lett. 31, 826-827 (1995)
18. Shih M.F., Leach P., Segev M., et al. Two-dimensional steady-state photorefractive screening solitons Opt.Lett. 21, 324-326 (1996)
19. Kos K., Meng H.X., Salamo G., et al. One-dimensional steady-state photorefractive screening solitons Phys.Rev.E 53, R4330-R4333 (1996)
20. DelRe E., Ciattoni A., Crosignani B., et al. Approach to space-charge field description in photorefractive crystals J.Opt.Soc.Am. B 15, 1469-1475 (1998)
21. Carvalho M.I, Singh S.R., Christodoulides D.N., Self-Deflection Of Steady-State Bright Spatial Solitons In Biased Photorefractive Crystals Opt.Commun. 120, 311-315 (1995)
22. Sheng Z.M., Cui Y.P., Cheng N., et al. Photorefractive self-trapping and deflection of optical beams J.Opt.Soc.Am. B 13, 584-589 (1996)
23. Carvalho M.I., Singh S.R., Christodoulides D.N., Self-deflection of steady-state bright spatial solitons in biased photorefractive crystals Opt.Commun. 124, 642-642 (1996)
24. Singh S.R., Carvalho M.I., Christodoulides D.N., Higher-order space charge field effects on the evolution of spatial solitons in biased photorefractive crystals Opt.Commun. 130, 288-294 (1996)
25. Krolikowski W., Akhmediev N., LutherDavies B., et al. Self-bending photorefractive solitons Phys.Rev.E 54, 5761-5765 (1996)
26. Liu J.S., Lu K.Q., Screening-photovoltaic spatial solitons in biased photovoltaic-photorefractive crystals and their self-deflection J.Opt.Soc.Am. B 16, 550-555 (1999)
27. Petter J., Weilnau C., Denz C., et al. Self-bending of photorefractive solitons Opt.Commun. 170, 291-297 (1999)
28. Aleshkevich V., Kartashov Y., Vysloukh V., Self-bending of the coupled spatial soliton pairs in a photorefractive medium with drift and diffusion nonlinearity Phys.Rev.E 63, 016603 (2001)
29. Liu J.S., Hao Z.H., Higher-order space-charge field effects on the self-deflection of bright screening photovoltaic spatial solitons J.Opt.Soc.Am. B 19, 513-521 (2002)
30. Montemezzani G., Gunter P., Profile of photorefractive one-dimensional bright spatial solitons Opt.Lett. 22, 451-453 (1997)
31. DelRe E., Agranat A.J., Dielectric nonlinearity in photorefractive spatial soliton formation Phys.Rev.A 65, 053814 (2002)
32. Kos K., Salamo G., Segev M., High-intensity nanosecond photorefractive spatial solitons Opt.Lett. 23, 1001-1003 (1998)
33. DelRe E., Crosignani B., Tamburrini M., et al. One-dimensional steady-state photorefractive spatial solitons in centrosymmetric paraelectric potassium lithium tantalate niobate Opt.Lett. 23, 421-423 (1998)
34. Lan S., Shih M.F., Segev M., Self-trapping of one-dimensional and two-dimensional optical beams and induced waveguides in photorefractive KNbO3 Opt.Lett. 22, 1467-1469 (1997)
35. Chauvet M., Hawkins S.A., Salamo G.J., et al. Self-trapping of two-dimensional optical beams and light-induced waveguiding in photorefractive InP at telecommunication wavelengths Appl.Phys.Lett. 70, 2499-2501 (1997)
36. DelRe E., Tamburrini M., Segev M., et al. Two-dimensional photorefractive spatial solitons in centrosymmetric paraelectric potassium-lithium-tantalate-niobate Appl.Phys.Lett. 73, 16-18 (1998)

37. Fazio E., Ramadan W., Belardini A., et al. (2+1)-dimensional soliton formation in photorefractive Bi12SiO20 crystals Phys.Rev.E 67, 026611 (2003)
38. She W.L., Lee K.K., Lee W.K., Observation of two-dimensional bright photovoltaic spatial solitons Phys.Rev.Lett. 83, 3182-3185 (1999)
39. Carmon T., Anastassiou C., Lan S., et al. Observation of two-dimensional multimode solitons Opt.Lett. 25, 1113-1115 (2000)
40. Carmon T., Uzdin R., Pigier C., et al. Rotating propeller solitons Phys.Rev.Lett. 87, 143901 (2001)
41. Krolikowski W., Ostrovskaya E.A., Weilnau C., et al. Observation of dipole-mode vector solitons Phys.Rev.Lett. 85, 1424-1427 (2000)
42. Mitchell M., Chen Z.G., Shih M.F., et al. Self-trapping of partially spatially incoherent light Phys.Rev.Lett. 77, 490-493 (1996)
43. Mitchell M., Segev M., Self-trapping of incoherent white light NATURE 387, 880-883 (1997)
44. Duree G., Morin M., Salamo G., et al. Dark Photorefractive Spatial Solitons And Photorefractive Vortex Solitons Phys.Rev.Lett. 74, 1978-1981 (1995)
45. Chen Z.G., Shih M.F., Segev M., et al. Steady-state vortex-screening solitons formed in biased photorefractive media Opt.Lett. 22, 1751-1753 (1997)
46. Chen Z.G., Segev M., Wilson D.W., et al. Self-trapping of an optical vortex by use of the bulk photovoltaic effect Phys.Rev.Lett. 78, 2948-2951 (1997)
47. Chen Z.G., Mitchell M., Segev M., et al. Self-trapping of dark incoherent light beams SCIENCE 280, 889-892 (1998)
48. Shih M.F., Segev M., Salamo G., Three-dimensional spiraling of interacting spatial solitons Phys.Rev.Lett. 78,2551-2554 (1997)
49. Shih M.F., Segev M., Incoherent collisions between two-dimensional bright steady-state photorefractive spatial screening solitons Opt.Lett. 21, 1538-1540 (1996)
50. Krolikowski W., Holmstrom S.A., Fusion and birth of spatial solitons upon collision Opt.Lett. 22, 369-371 (1997)
51. Krolikowski W., Luther-Davies B., Denz C., et al. Annihilation of photorefractive solitons Opt.Lett. 23, 97-99 (1998)
52. Zozulya A.A., Anderson D.Z., Mamaev A.V., et al. Self-focusing and soliton formation in media with anisotropic nonlocal material response Europhys.Lett. 36, 419-424 (1996)
53. Gatz S., Herrmann J., Propagation of optical beams and the properties of two-dimensional spatial solitons in media with a local saturable nonlinear refractive index J.Opt.Soc.Am. B 14, 1795-1806 (1997)
54. Crosignani B., DiPorto P., Degasperis A., et al. Three-dimensional optical beam propagation and solitons in photorefractive crystals J.Opt.Soc.Am. B 14, 3078-3090 (1997)
55. Gatz S., Herrmann J., Anisotropy, nonlocality, and space-charge field displacement in (2+1)-dimensional self-trapping in biased photorefractive crystals Opt.Lett. 23, 1176-1178 (1998)
56. Belic M.R., Vujic D., Stepken A., et al. Isotropic versus anisotropic modeling of photorefractive solitons Phys.Rev.E 65, 066610 (2002)
57. Calvo G.F., Agullo-Lopez F., Carrascosa M., et al. Two-dimensional soliton-induced refractive index change in photorefractive crystals Opt.Commun. 227, 193-202 (2003)

58. Meng H.X., Salamo G., Segev M., Primarily isotropic nature of photorefractive screening solitons and the interactions between them Opt.Lett. 23, 897-899 (1998)
59. Krolikowski W., Saffman M., Luther-Davies B., et al. Anomalous interaction of spatial solitons in photorefractive media Phys.Rev.Lett. 80, 3240-3243 (1998)
60. DelRe E., Ciattoni A., Agranat A.J., Anisotropic charge displacement supporting isolated photorefractive optical needles Opt.Lett. 26, 908-910 (2001)
61. Mamaev A.V., Saffman M., Anderson D.Z., et al. Propagation of light beams in anisotropic nonlinear media: From symmetry breaking to spatial turbulence Phys.Rev.A 54, 870-879 (1996)
62. Shih M.F., Jeng C.C., Sheu F.W., et al. Spatiotemporal optical modulation instability of coherent light in noninstantaneous nonlinear media Phys.Rev.Lett. 88, 133902 (2002)
63. C. Dari-Salisburgo, E. DelRe, and E. Palange Molding and stretched evolution of optical solitons in cumulative nonlinearities Phys.Rev.Lett. 91, 263903 (2003)
64. Fressengeas N., Maufoy J., Kugel G., Temporal behavior of bidimensional photorefractive bright spatial solitons Phys.Rev.E 54, 6866-6875 (1996)
65. Fressengeas N., Maufoy J., Wolfersberger D., et al. Experimental transient self-focusing in Bi12TiO20 crystal Ferroelectrics 202, 193-202 (1997)
66. Fressengeas N., Wolfersberger D., Maufoy J., et al. Build up mechanisms of (1+1)-dimensional photorefractive bright spatial quasi-steady-state and screening solitons Opt.Commun 145, 393-400 (1998)
67. Fressengeas N., Wolfersberger D., Maufoy J., et al. Experimental study of the self-focusing process temporal behavior in photorefractive Bi12TiO20 J.Appl.Phys 85, 2062-2067 (1999)
68. Wolfersberger D., Fressengeas N., Maufoy J., et al. Experimental study of the behaviour of narrow nanosecond laser pulses in biased photorefractive Bi12TiO20 Ferroelectrics 223, 381-388 (1999)
69. Maufoy J., Fressengeas N., Wolfersberger D., et al. Simulation of the temporal behavior of soliton propagation in photorefractive media Phys.Rev.E 59, 6116-6121 (1999)
70. Denz C., Krolikowski W., Petter J., et al. Dynamics of formation and interaction of photorefractive screening solitons Phys.Rev.E 60, 6222-6225 (1999)
71. Tosi-Beleffi G.M., Curti F., Boschi D., et al. Soliton-based Y-branch in photorefractive crystals induced through dispersion-shifted optical fiber Opt.Lett. 28, 1561-1563 (2003)
72. Fuentes-Hernandez C.A., Khomenko A.V., Beam collapse and polarization self-modulation in an optically active photorefractive crystal in an alternating electric field Phys.Rev.Lett. 83, 1143-1146 (1999)
73. Frolova M.N., Shandarov S.M., Borodin M.V. Self-action of a light beam in a photorefractive crystal in an alternating electric field upon synchronous intensity modulation Quantum.Electron. 32, 45-48 (2002)
74. Wolfersberger D., Fressengeas N., Maufoy J., et al. Self-focusing of a single laser pulse in a photorefractive medium Phys.Rev.E 62, 8700-8704 (2000)
75. Wolfersberger D., Lhomme F., Fressengeas N., et al. Simulation of the temporal behavior of one single laser pulse in a photorefractive medium Opt.Commun. 222, 383-391 (2003)
76. Valley G.C., Segev M., Crosignani B., Yariv A., Fejer M.M., and Bashaw M., Dark and bright photovoltaic spatial solitons. Phys.Rev.A **50**, R4457-R4460 (1994)

77. Crosignani B., DelRe E., Di Porto P., et al. Self-focusing and self-trapping in unbiased centrosymmetric photorefractive media Opt.Lett. 23, 912-914 (1998)
78. Chauvet M., Hawkins S.A., Salamo G.J., et al. Self-trapping of planar optical beams by use of the photorefractive effect in InP:Fe Opt.Lett. 21, 1333-1335 (1996)
79. DelRe E., Tamburrini M., Segev M., et al. Spontaneous self-trapping of optical beams in metastable paraelectric crystals Phys.Rev.Lett. 83, 1954-1957 (1999)
80. Klotz M., Meng H.X., Salamo G.J., et al. Fixing the photorefractive soliton Opt.Lett. 24, 77-79 (1999)
81. Klotz M., Crosser M., Guo A., et al. Fixing solitonic y junctions in photorefractive strontium-barium-niobate Appl.Phys.Lett. 79, 1423-1425 (2001)
82. Segev M., Valley G.C., Bashaw M.C., et al. Photovoltaic spatial solitons J.Opt.Soc.Am. B 14, 1772-1781 (1997)
83. Taya M., Bashaw M.C., Fejer M.M., Segev M. and Valley G.C., Observation of dark photovoltaic spatial solitons. Phys.Rev.A **52**, 3095-3100 (1995)
84. Anastassiou C., Shih M.F., Mitchell M., et al. Optically induced photovoltaic self-defocusing-to-self-focusing transition Opt.Lett. 23, 924-926 (1998)
85. Schwartz T., Ganor Y., Carmon T., et al. Photorefractive solitons and light-induced resonance control in semiconductor CdZnTe Opt.Lett. 27, 1229-1231 (2002)
86. Uzdin R., Segev M., Salamo G.J., Theory of self-focusing in photorefractive InP Opt.Lett. 26, 1547-1549 (2001)
87. Crosignani B., Degasperis A., DelRe E., et al. Nonlinear optical diffraction effects and solitons due to anisotropic charge-diffusion-based self-interaction Phys.Rev.Lett. 82, 1664-1667 (1999)
88. Shih M.F., Sheu F.W., Photorefractive polymeric optical spatial solitons Opt.Lett. 24, 1853-1855 (1999)
89. Sheu F.W., Shih M.F., Photorefractive polymeric solitons supported by orientationally enhanced birefringent and electro-optic effects J.Opt.Soc.Am. B 18, 785-793 (2001)
90. Chen Z.G., Asaro M., Ostroverkhova O., et al. Self-trapping of light in an organic photorefractive glass Opt.Lett. 28, 2509-2511 (2003)
91. Castillo M.D., Sanchezmondragon J.J., Stepanov S.I., et al. Probe Beam Wave-Guiding Induced By Spatial Dark Solitons In Photorefractive Bi12TiO2 Crystal Rev.Mex.Fis 41, 1-10 (1995)
92. Castillo M.D., Sanchezmondragon J.J., Stepanov S.I., et al. (1+1)-Dimension Dark Spatial Solitons In Photorefractive Bi12TiO2 Crystal.Opt.Commun. 118, 515-519 (1995)
93. Sarabia C.M.G., Aguilar P.A.M., Mondragon J.J.S., et al. Dynamics of photoinduced lens formation in a photorefractive Bi12TiO2 crystal under an external dc electric field J.Opt.Soc.Am. B 13, 2767-2774 (1996)
94. Singh S.R., Christodoulides D.N., Effects of optical activity on photorefractive spatial solitons in a biased Bi12TiO2 crystal J.Opt.Soc.Am. B 13, 719-724 (1996)
95. Krolikowski W., Akhmediev N., Andersen D.R., et al. Effect of natural optical activity on the propagation of photorefractive solitons Opt.Commun. 132, 179-189 (1996)
96. Fazio E., Babin V., Bertolotti M., et al. Solitonlike propagation in photorefractive crystals with large optical activity and absorption Phys.Rev.E 66, 016605 (2002)

97. Segev M., Agranat A.J., Spatial solitons in centrosymmetric photorefractive media Opt.Lett. 22, 1299-1301 (1997)
98. Shih M.F., Chen Z.G., Segev M., et al. Incoherent collisions between one-dimensional steady-state photorefractive screening solitons Appl.Phys.Lett. 69, 4151-4153 (1996)
99. Meng H.X., Salamo G., Shih M.F., et al. Coherent collisions of photorefractive solitons Opt.Lett. 22, 448-450 (1997)
100. Mamaev A.V., Saffman M., Zozulya A.A., Phase-dependent collisions of (2+1)-dimensional spatial solitons J.Opt.Soc.Am. B 15, 2079-2082 (1998)
101. Buryak A.V., Kivshar Y.S., Shih M.F., et al. Induced coherence and stable soliton spiraling Phys.Rev.Lett. 82, 81-84 (1999)
102. Belic M.R., Stepken A., Kaiser F., Spiraling behavior of photorefractive screening solitons Phys.Rev.Lett. 82, 544-547 (1999)
103. Stepken A., Belic M.R., Kaiser F., et al. Three dimensional trajectories of interacting incoherent photorefractive solitons Phys.Rev.Lett. 82, 540-543 (1999)
104. DelRe E., Trillo S., Agranat A.J., Collisions and inhomogeneous forces between solitons of different dimensionality Opt.Lett. 25, 560-562 (2000)
105. DelRe E., Ciattoni A., Crosignani B., et al. Nonlinear optical propagation phenomena in near-transition centrosymmetric photorefractive crystals J.Nonlinear.Opt.Phys. 8, 1-20 (1999)
106. Kip D., Herden C., Wesner M., All-optical signal routing using interaction of mutually incoherent spatial solitons Ferroelectrics 274, 135-142 (2002)
107. Belic M., Jander P., Strinic A., et al. Self-trapped bidirectional waveguides in a saturable photorefractive medium Phys.Rev.E 68, 025601 (2003)
108. Rotschild C., Cohen O., Manela O., Carmon T., and Segev M., Interactions between spatial screening solitons propagating in opposite directions, to be publsihed, Journal of the Optical Society of America B, 2004
109. Cohen O., Uzdin R., Carmon T., Fleischer J.W., Segev M., and Odoulov S., Collisions between Optical Spatial Solitons Propagating in Opposite Directions Phys.Rev.Lett. 89, 133901 (2002)
110. Cohen O., Lan S., Carmon T., Giordmaine J. A., and Segev M., Spatial vector solitons consisting of counter-propagating fields, Opt.Lett. 27, 2013 (2002)
111. Christodoulides D.N., Singh S.R., Carvalho M.I., et al. Incoherently coupled soliton pairs in biased photorefractive crystals Appl.Phys.Lett. 68, 1763-1765 (1996)
112. Chen Z.G., Segev M., Coskun T.H., et al. Observation of incoherently coupled photorefractive spatial soliton pairs Opt.Lett. 21, 1436-1438 (1996)
113. Chen Z., Segev M., Coskun T., Christodoulides D., Kivshar Y., and Afanasjev V., Observation of incoherently coupled dark-bright photorefractive spatial soliton pairs, Opt. Lett. 21, 1821 (1996)
114. Mitchell M., Segev M. and Christodoulides D. N., Observation of multi-hump multi-mode solitons Phys.Rev.Lett. 80, 4657-4660 (1998)
115. Pigier C., Uzdin R., Carmon T., Segev M., Nepomnyashchy A. and Musslimani Z. H., Collisions between (2+1)D rotating propeller solitons, Opt.Lett. 26, 1577 (2001)
116. Anastassiou C., Segev M., Steiglitz K., et al. Energy-exchange interactions between colliding vector solitons Phys.Rev.Lett. 83, 2332-2335 (1999)
117. Anastassiou C., Fleischer J. W., Carmon T., Segev M., and Steiglitz K., Information transfer through cascaded collisions of vector solitons, Optics Letters 26, 1498 (2001)

118. Jakubowski M. H., Steiglitz K. and Squier R., Phys. Rev. E 58, 6752 (1998)
119. Krolikowski W., Akhmediev N., Luther-Davies B., Collision-induced shape transformations of partially coherent solitons Phys.Rev.E 59, 4654-4658 (1999)
120. Christodoulides D.N., Coskun T.H., Mitchell M., et al. Theory of incoherent self-focusing in biased photorefractive media Phys.Rev.Lett. 78, 646-649 (1997)
121. Mitchell M., Segev M., Coskun T.H., et al. Theory of self-trapped spatially incoherent light beams Phys.Rev.Lett. 79, 4990-4993 (1997)
122. Shkunov V.V. and Anderson D.Z., Radiation Transfer Model of Self-Trapping Spatially Incoherent Radiation by Nonlinear Media Phys.Rev.Lett. 81, 2683-2686 (1998)
123. Christodoulides D.N., Eugenieva E.D., Coskun T.H., et al. Equivalence of three approaches describing partially incoherent wave propagation in inertial nonlinear media Phys.Rev.E 63, 035601 (2001)
124. Buljan H., Segev M., Soljacic M., Efremidis N.K., and Christodoulides D.N., White light solitons, Opt.Lett. 28, 1239-1241 (2003)
125. Buljan H., Siber A., Soljacic M., Schwartz T., Segev M., and Christodoulides D.N., White light solitons in logarithmically saturable nonlinear media Phys.Rev. E 68, 036607 (2003)
126. Coskun T.H., Christodoulides D.N., Kim Y.R., et al. Bright spatial solitons on a partially incoherent background Phys.Rev.Lett. 84, 2374-2377 (2000)
127. Eugenieva E.D., Christodoulides D.N., Segev M., Elliptic incoherent solitons in saturable nonlinear media Opt.Lett. 25, 972-974 (2000)
128. Katz O., Carmon T., Schwartz T., Segev M. and Christodoulides D.N., Observation of elliptic incoherent spatial solitons, to be published in Opt.Lett. 2004
129. Coskun T.H., Grandpierre A.G., Christodoulides D.N., et al. Coherence enhancement of spatially incoherent light beams through soliton interactions Opt.Lett. 25, 826-828 (2000)
130. Coskun T.H., Christodoulides D.N., Mitchell M., et al. Dynamics of incoherent bright and dark self-trapped beams and their coherence properties in photorefractive crystals Opt.Lett. 23, 418-420 (1998)
131. Soljacic M., Segev M., Coskun T., et al. Modulation instability of incoherent beams in noninstantaneous nonlinear media Phys.Rev.Lett. 84, 467-470 (2000)
132. Kip D., Soljacic M., Segev M., Eugenieva E., and Christodoulides D.N., Modulation instability and pattern formation in spatially incoherent light beams Science 290, 495 (2000)
133. Kip D., Soljacic M., Segev M., et al. (1+1)-Dimensional modulation instability of spatially incoherent light J.Opt.Soc.Am. B 19, 502-512 (2002)
134. Anastassiou C., Soljacic M., Segev M., Kip D., Eugenieva E., Christodoulides D.N. and Musslimani Z.H., Eliminating the transverse instabilities of Kerr solitons, Phys.Rev.Lett. 85, 4888-4891 (2000)
135. Torres J.P., Anastassiou C., Segev M., Soljacic M. and Christodoulides D.N., Transverse instability of incoherent solitons in Kerr media, Phys.Rev. E 65, 15601 (2001)
136. Jeng C.C., Shih M., Motzek K., Kivshar Y., Partially incoherent optical vortices in self-focusing nonlinear media, Phys.Rev.Lett. 92, 043904 (2004)
137. Chen Z., Sears S.M., Martin H., Christodoulides D.N. and Segev M., Clustering of solitons in weakly correlated systems, Proceedings of the US National Academy of Science (PNAS) 99, 5223 (2002)

138. Sears S.M., Soljacic M., Christodoulides D.N. and Segev M., Pattern formation via symmetry breaking in nonlinear weakly correlated systems Phys.Rev. E 65, 36620 (2002)
139. Buljan H., Siber A., Soljacic M., and Segev M., Propagation of incoherent "white" light and modulation instability in non-instantaneous nonlinear media Phys.Rev. E 66, R35601 (2002)
140. Schwartz T., Buljan H., and Segev M., Modulation instability and pattern formation with white incoherent light, submitted for publication
141. Carmon T., Soljacic M., and Segev M., Pattern formation in a cavity longer than the coherence length of the light in it, Phys.Rev.Lett. 89, 183902 (2002)
142. Schwartz T., Fleischer J.W., Carmon T., Cohen O., Buljan H., and Segev M., Pattern formation in a ring cavity with a temporally-incoherent feedback, submitted to the Journal of the Optical Society of America B, January 2004
143. Buljan H., Soljacic M., Carmon T., and Segev M., Cavity pattern formation with incoherent light, Phys.Rev. E 68, 016616 (2003)
144. Carmon T., Buljan H., and Segev M., Spontaneous pattern formation in a cavity with incoherent light, submitted
145. Morin M., Duree G., Salamo G., et al. Wave-Guides Formed By Quasi-Steady-State Photorefractive Spatial Solitons Opt.Lett. 20, 2066-2068 (1995)
146. Torrescisneros G., Sanchezmondragon J., Iturbecastillo M., et al. Optical Wave-Guides Generated By Spatial Solitons Rev.Mex.Fis 41, 662-694 (1995)
147. Shih M.F., Segev M., Salamo G., Circular waveguides induced by two-dimensional bright steady-state photorefractive spatial screening solitons Opt.Lett. 21, 931-933 (1996)
148. Liu S.M., Zhang G.Q., Sun Q., et al. Waveguides written and stored by photovoltaic dark spatial solitons in $LiNbO_3$:Fe crystals Chin.Phys.Lett. 13, 737-740 (1996)
149. Shih M.F., Chen Z.G., Mitchell M., et al. Waveguides induced by photorefractive screening solitons J.Opt.Soc.Am. B 14, 3091-3101 (1997)
150. Petter J., Denz C., Stepken A., et al. Anisotropic waveguides induced by photoreftactive (2+1)D solitons J.Opt.Soc.Am. B 19, 1145-1149 (2002)
151. Kip D., Anastassiou C., Eugenieva E., et al. Transmission of images through highly nonlinear media by gradient-index lenses formed by incoherent solitons Opt.Lett. 26, 524-526 (2001)
152. Lan S., DelRe E., Chen Z.G., et al. Directional coupler with soliton-induced waveguides Opt.Lett. 24, 475-477 (1999)
153. Chen Z.G., Mitchell M., Segev M., Steady-state photorefractive soliton-induced Y-junction waveguides and high-order dark spatial solitons Opt.Lett. 21 (10): 716-718 MAY 15 (1996)
154. Taya M., Bashaw M.C., Fejer M.M., et al. Y junctions arising from dark-soliton propagation in photovoltaic media Opt.Lett. 21, 943-945 (1996)
155. Chen Z.G., Segev M., Singh S.R., et al. Sequential formation of multiple dark photorefractive spatial solitons: Experiments and theory J.Opt.Soc.Am. B 14, 1407-1417 (1997)
156. Kip D., Wesner M., Shandarov V., et al. Observation of bright spatial photorefractive solitons in a planar strontium barium niobate waveguide Opt.Lett. 23, 921-923 (1998)
157. Wesner M., Herden C., Kip D., et al. Photorefractive steady state solitons up to telecommunication wavelengths in planar SBN waveguides Opt.Commun. 188, 69-76 (2001)

158. Chauvet M., Chauvin S., Maillotte H., Transient dark photovoltaic spatial solitons and induced guiding in slab LiNbO3 waveguides Opt.Lett. 26, 1344-1346 (2001)
159. Guo A., Henry M., Salamo G.J., et al. Fixing multiple waveguides induced by photorefractive solitons: directional couplers and beam splitters Opt.Lett. 26, 1274-1276 (2001)
160. Eugenieva E.D., Roussev R.V., Dinev S.G., Soliton-controlled switching in photorefractive media J.Mod.Optic. 44, 1127-1140 (1997)
161. Andrade-Lucio J.A., Mendez-Otero M.M., Gomez-Sarabia C.M., et al. Controllable optical Y-junctions based on dark spatial solitons generated by holographic masks Opt.Commun. 165, 77-82 (1999)
162. Malomed B.A., Multichannel switchable system for spatial solitons J.Opt.Soc.Am. B 16, 1197-1203 (1999)
163. Dittrich P., Montemezzani G., Bernasconi P., et al. Fast, reconfigurable light-induced waveguides Opt.Lett. 24, 1508-1510 (1999)
164. Andrade-Lucio J.A., Alvarado-Mendez B., Rojas-Laguna R., et al. Optical switching by coherent collision of spatial solitons Electron.Lett. 36, 1403-1405 (2000)
165. Petter J., Denz C., Guiding and dividing waves with photorefractive solitons Opt.Commun. 188, 55-61 (2001)
166. DelRe E., Tamburrini M., Agranat A.J., Soliton electro-optic effects in paraelectrics Opt.Lett. 25, 963-965 (2000)
167. DelRe E., Crosignani B., Di Porto P., et al. Electro-optic beam manipulation through photorefractive needles Opt.Lett. 27, 2188-2190 (2002)
168. Lan S., Shih M.F., Mizell G., et al. Second-harmonic generation in waveguides induced by photorefractive spatial solitons Opt.Lett. 24, 1145-1147 (1999)
169. Lan S., Anastassiou C., Segev M., et al. Tuning of second-harmonic generation in waveguides induced by photorefractive spatial solitons Appl.Phys.Lett 77, 2101-2103 (2000)
170. Boardman A.D., Ilecki W., Liu Y., Spatial solitons in a photorefractive medium sustaining second-harmonic generation J.Opt.Soc.Am. B 19, 832-838 (2002)
171. Lan S., Giordmaine J.A., Segev M., et al. Optical parametric oscillation in soliton-induced waveguides Opt.Lett. 27, 737-739 (2002)
172. Efremidis N.K., Sears S., Christodoulides D.N., et al. Discrete solitons in photorefractive optically induced photonic lattices Phys.Rev. E 66, 046602 (2002)
173. Fleischer J.W., Carmon T., Segev M., et al. Observation of discrete solitons in optically induced real time waveguide arrays Phys.Rev.Lett. 90, 023902 (2003)
174. Fleischer J. W., Segev M., Efremidis N. K., and Christodoulides D. N., Observation of two-dimensional discrete solitons in optically-induced nonlinear photonic lattices Nature 422, 147 (2003).
175. Petter J., Schroder J., Trager D., et al. Optical control of arrays of photorefractive screening solitons Opt.Lett. 28, 438-440 (2003)
176. Neshev D., Ostrovskaya E., Kivshar Y., et al. Spatial solitons in optically induced gratings Opt.Lett. 28, 710-712 (2003)
177. Petrovic M., Trager D., Strinic A., et al. Solitonic lattices in photorefractive crystals Phys.Rev.E 68, 055601 (2003)
178. Anastassiou C., Pigier C., Segev M., et al. Self-trapping of bright rings Opt.Lett. 26, 911-913 (2001)